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# Shape reconstruction of delamination defects using thermographic infrared signals based on an enhanced Canny approach

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## Abstract

The Canny shape-reconstruction algorithm is combined with the thermographic signal reconstruction (TSR) technique for the characterisation of delamination defects in planar multilayer structures. The adopted approach consists in fitting the raw thermographic images to a low-degree polynomial in the log-log representation of the time axis, according to TSR technique, and then apply the Canny algorithm to the reconstruction of the original signals. The proposed juxtaposition of the two treatments benefits from the inherent signal de-noising and compression of the TSR algorithm, thus enhancing the performance of the overall reconstruction process. The proposed approach has been applied using synthetic signals obtained via a 3D semi-analytical simulation code and the obtained reconstructed images have been compared with the (known) delamination geometry demonstrating satisfactory results.

*Keywords:* infrared testing, delamination, image reconstruction, Canny algorithm, TSR, semi-analytical solution.

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## 1. Introduction

Infrared non-destructive testing (IR) techniques have received growing attention in recent years thanks to their advantages, that is, fast, high resolution and contactless control, and benefiting from the technological progress in the domain of the hardware, namely infrared cameras and data acquisition equipment. The acquired infrared images are then analysed for the detection and characterization of flaws. Subsurface irregularities will affect the heat diffusion rate leading to a thermal contrast on the surface of the homogeneous work-piece. By analysing alterations or the contrast in the thermal pattern of the material surface, one can obtain information about subsurface flaws.

The material under testing may be heated using various sources as optical excitation, electromagnetic excitation, acoustic excitation, stress excitation. The most common form of active thermography [1] for material evaluation consists in using sources as flash lamps [2] or lasers [3, 4] where a pulse of light instantaneously heats a surface and the resulting temperature change is observed with a thermal camera. This technique is also known as pulsed thermography (PT) [5] or square pulse thermography (SPT) [6] and has been extensively used as inspection technology for composites and layered structures. According to the heating function, other common types of active thermography are stepped thermography (ST) [7], lock-in thermography (LT) [8, 9], pulsed phase thermography (PPT) [10], and frequency modulated thermography (FMT) [11].

The detection of the defects can be on the same side as the heat source or on the opposite side, depending on the type of access to the sample. Typical TNDT procedure re-

sults in a sequence of infrared (IR) images, obtained via an IR camera, that reflects the evolution of temperature over time. The recorded thermal response usually is degraded because of several factors. Uneven heating and variations of emissivity on the observed surface are only few of the factors which can cause signal-to-noise-ratio (SNR) problems and limit the potential sensitivity of any method.

Analysing the thermal images may be an intense and not an easy task to perform. The detection and characterization of the defects highly depends on the quality of the images as well as on the techniques used. Signal processing is a crucial mean to extract useful information from raw data captured from sensors. More and more signal processing algorithms including thermographic signal reconstruction (TSR) [12, 13], principal component analysis (PCA) [14], independent component analysis (ICA) [15, 16], wavelet transform [17], Tucker decomposition [18, 19], support vector machine (SVM) [20], and pattern recognition [21] are being used in thermography. The use of any of the aforementioned techniques depends on the physics of the problem and their results are usually qualitative, meaning that the defects are not fully characterised. Depending on the application, the detection and the shape reconstruction of the defects may be a success.

In the present work, the TSR method has been used to reduce the amount of noise in thermal images and local storage requirements while improving the visibility of defects. The technique significantly reduces temporal noise. Even though the TSR is a pixel-based technique, one of its side effects is the reduction of spatial noise, making the images more suitable for the defects' detection.

Underlying defects and discontinuities appear explic-

64 itly in the reconstructed images with a different intensity.121  
65 Morphological image processing techniques can be used to122  
66 detect these abnormalities. Almond *et al.* here [22] sug-123  
67 gested the identification procedure called Full Width Half124  
68 Maximum (FWHM) which is based on the measurement of125  
69 the distance between the points which correspond to half of126  
70 the maximum temperature signal. Thermal edge effects for127  
71 crack-like defects have been calculated using the Wiener-128  
72 Hopf technique. Lugin and Netzelmann [23] proposed a129  
73 model-based iterative algorithm for the reconstruction of130  
74 two- and three-dimensional defect shapes.131

75 More generic image processing techniques can be used132  
76 for the detection of the defects' edges. A suitable edge-133  
77 detection technique which can be used for accurate recog-134  
78 nition and shape reconstruction is the Canny algorithm [24]135  
79 Canny edge detector is a multi-step technique that pro-136  
80 cesses an input image and detects the edges of any objects.137  
81 Because of its great accuracy, the algorithm is used in com-138  
82 puter vision systems, yet not only. In the NDT field the139  
83 applications of the algorithm are numerous and its usage140  
84 can be found in several inspection techniques. Alaknanda141  
85 *et al.* [25] used the Canny algorithm for the flaw detection142  
86 in radiographic weld images. In a recent work by Yan *et al.*143  
87 [26] the algorithm has been used for the detection of144  
88 corrosion via pulse-modulation eddy current inspection.145  
89 In thermography, Sham *et al.* [27] used the algorithm for146  
90 the detection of cracks on concrete surfaces heated by a147  
91 flash lamp.148

92 This paper proposes a new approach for the real-time149  
93 shape reconstruction of delamination defects in planar mul-150  
94 tilayer structures based on a combination of the TSR method151  
95 and the Canny algorithm. To assess the performance of152  
96 the approach, the proposed algorithm is applied to syn-153  
97 thetic thermograms obtained by corrupting simulation re-  
98 sults with different levels of additive Gaussian noise.

99 The numerical simulation of IR procedures involves154  
100 the solution of the heat conduction problem in the con-155  
101 sidered work-piece, with and without defects, in order to156  
102 obtain the temperature distribution at the piece interfaces,  
103 which constitutes the measurement. The full solution to  
104 this problem can be obtained using a numerical technique  
105 like the finite elements method (FEM). Nevertheless, in  
106 practical situations it is often meaningful to renounce the  
107 detailed information of the complete numerical solution in  
108 favour of fast analytical or semi-analytical approximations,  
109 which hold the essence of the thermal flow behaviour. A  
110 powerful, well-established approach is the so-called ther-  
111 mal quadrupole method, where the original three dimen-  
112 sional problem is approximated as a multilayer one dimen-  
113 sional problem (by ignoring the heat flow in the lateral lay-  
114 ers directions) and modelled as a cascade of "quadrupoles"  
115 in analogy with the electrical network theory [28, 29, 30].  
116

117 Should the lateral propagation become no longer neg-171  
118 ligible, as in the case of thermal image processing at late  
119 times considered in this work, the quadrupole approach  
120 is not valid anymore, and a more general approach prop-  
erly accounting for the 3D diffusion should be followed.

Semi-analytical formulations are still tractable in this case,  
provided that the considered geometry consists of canon-  
ical pieces and making the assumption that the solution  
is negligible (or stationary) at sufficient distance from the  
region of interest. This approach is based on the artificial  
truncation of the computational domain, referred to in the  
literature as the truncated region eigenfunction expansion  
(TREE), and has been successfully applied in electromag-  
netics for the solution of magnetostatic and low-frequency  
(eddy-current) problems [31, 32, 33, 34, 35]. Defects can  
be modelled either directly as part of the geometry like  
in [33, 34, 35], or indirectly by first applying the TREE  
method to construct the Green's function accounting for  
the geometry of the flawless piece and treating the defect  
as a perturbation by solving the appropriate integral equa-  
tion [36, 37].

The TREE approach has already been tested for the  
inversion of thermographic signals in order to estimate the  
thickness of delamination defects [38]. In this work, this  
approach is extended to multilayer specimens and is com-  
bined with the TSR method for providing the simulated  
signals to the reconstruction algorithm.

The paper is organised as follows. The considered in-  
spection problem is posed and a semi-analytical solution  
of the heat conduction problem using the TREE approach  
is presented in section 2. The defect reconstruction al-  
gorithm is then discussed in section 3, where a detailed  
presentation of the constituting bricks, namely the TSR  
and the Canny algorithm, is provided. The application of  
the proposed approach in the case of a two-layer medium  
embedding a number of delamination patches between the  
two layers is examined in section 4, and the article is con-  
cluded with a brief discussion in section 5.

## 2. Heat propagation problem in multilayer pieces with delamination defects

### 2.1. Problem position

Let us consider a planar piece composed of  $N$  stacked  
sheets perfectly joined with another. The medium above  
and underneath the considered piece is air. The piece is  
thermally excited at its upper interface by means of an im-  
pinging heat flux (flash lamp, laser source, air flow). The  
exact form of the excitation is irrelevant for the hereafter  
developed mathematical analysis. Solely its geometrical  
features, i.e., the shape and the dimensions of the flux  
spot as well as its intensity, that is, the heat power per  
unit of surface, are of interest here. We also assume that  
the considered medium is affected by a delamination flaw  
located between the  $i$ th and the  $(i + 1)$ th layer. As such,  
it is understood a local loss of contact between the two  
layers, which physically takes the form of a very thin cav-  
ity between the two layers, filled with air. The considered  
configuration is depicted in Fig. 1.

Each layer  $i$  is characterised by its mass density  $\rho_i$ , its  
heat capacity at constant (atmospheric) pressure  $c_{p_i}$  and

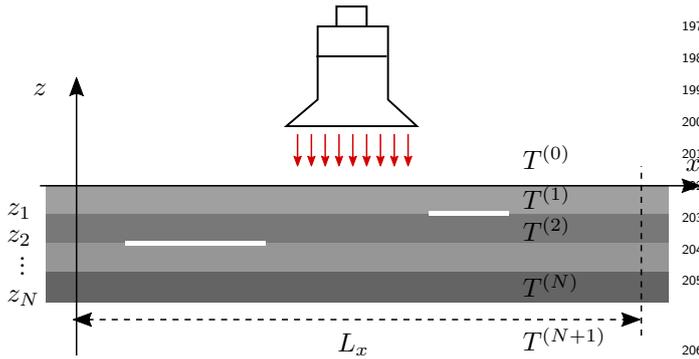


Figure 1: Problem configuration: a planar multilayer medium affected by a number of delamination flaws is thermally excited by an impinging heat flux.

its thermal conductivity, which in the context of this work we assume that it is isotropic  $\kappa_i$ .

We seek to calculate the temperature at every point of the medium throughout an observation time window comprising the excitation and a relaxation time interval after the source switch-off. In particular, we are interested in the temperature distribution at the upper and lower piece-air interface since these distributions are the experimentally accessible information. These two distributions can be extracted by the solution of the associated heat propagation problem, as developed below.

## 2.2. Formal solution

Let  $T^{(i)}(\mathbf{r}, t)$  be the temperature distribution inside the  $i$ th layer, as shown in Fig. 1, expressed as a function of position  $\mathbf{r}$  and time  $t$ . Since no heat sources exist inside the layer by hypothesis,  $T^{(i)}(\mathbf{r}, t)$  satisfies the homogeneous heat equation in this layer

$$\left( \nabla^2 - \frac{1}{\alpha_i} \frac{\partial}{\partial t} \right) T^{(i)}(\mathbf{r}, t) = 0, \quad (1)$$

where  $\alpha_i = \kappa_i / c_p \rho_i$  stands for the diffusivity of the layer.

Since we are interested in the transient solution of the heat equation, and we are particularly keen to the early time response, the established (and also most efficient) approach is to consider the problem in Laplace domain. Application of the Laplace transform to (1) yields

$$\left( \nabla^2 - \frac{s}{\alpha_i} \right) T^{(i)}(\mathbf{r}, s) = 0, \quad (2)$$

$s$  being the Laplace variable.

In media with flaws the total solution for the temperature field can be decomposed into two terms: the solution for the unperturbed medium under the excitation of the considered heat source, and a perturbation term produced by the interaction of the incoming heat flux with the material flaw. Given that we are dealing with a diffusion problem, the latter contribution can be considered as negligible at sufficiently remote distance from the flaw, in the same

way we treat the flaw field in eddy current problems. As a consequence, the temperature field at these distances will be dominated by the solution for the unperturbed medium, which for an infinite planar piece depends solely on the dimensions of the incoming heat flux. Hence, for extended (practically infinite) fluxes, the computational domain can be closed (truncated) by a Neumann boundary condition, which assures zero escaping heat flux from the domain boundaries (otherwise stated, an adiabatic condition).

$$\frac{\partial T^{(i)}}{\partial x} \Big|_{x=0} = \frac{\partial T^{(i)}}{\partial x} \Big|_{x=L_x} = 0, \quad (3)$$

$$\frac{\partial T^{(i)}}{\partial y} \Big|_{y=0} = \frac{\partial T^{(i)}}{\partial y} \Big|_{y=L_y} = 0, \quad (4)$$

with the computational domain extending from 0 to  $L_x$  in the  $x$  direction and from 0 to  $L_y$  in the  $y$  direction. This condition is in accordance with the physics of the solution. If, instead, the consider source has a finite (with respect to the characteristic dimensions of the problem) support, the domain can be truncated either by a Dirichlet (zero temperature) or Neumann (zero flux) condition, as it is the usual practice with the TREE method<sup>1</sup>.

In this work we deliberately choose to work with finite support sources since this case leads to simpler mathematical treatment (absence of a zero-order term). It does not lack practical significance either, since narrow lamp spots and point sources like laser beams are also of interest in real world applications. We thus impose

$$T^{(i)} \Big|_{x=0} = T^{(i)} \Big|_{x=L_x} = 0, \quad (5)$$

$$T^{(i)} \Big|_{y=0} = T^{(i)} \Big|_{y=L_y} = 0. \quad (6)$$

Notice here that re-gauging the temperature field to be zero at the ambient conditions, we can apply the homogeneous Dirichlet condition at the domain boundaries. The formal solution for the temperature field in the  $i$ th layer reads as

$$T^{(i)} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn}^{(i)} e^{\eta_{mn}^{(i)}(z-z_{i-1})} w_{mn}(x, y) + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} D_{mn}^{(i)} e^{-\eta_{mn}^{(i)}(z-z_i)} w_{mn}(x, y) \quad (7)$$

for  $i = 1 \dots, N$ , with

$$w_{mn}(x, y) = \sin(\kappa_m x) \sin(\lambda_n y), \quad (8)$$

the discrete eigenvalues  $\kappa_m$  and  $\lambda_n$  being determined by the truncation conditions in the  $x$  and  $y$  directions (5),(6),

<sup>1</sup>It should be underlined here the difference with the eddy-current problems, where the TREE formulation has been extensively studied. In these problems, the excitation term is always localised around the coil and hence one can use either Dirichlet or Neumann conditions in almost every case. The basic criterion in the final choice is mathematical convenience.

namely,

$$\sin(\kappa_m L_x) = 0, \quad \kappa_m = \frac{m\pi}{L_x}, \quad m = 1 \dots, \infty, \quad (9)$$

$$\sin(\lambda_n L_y) = 0, \quad \lambda_n = \frac{n\pi}{L_y}, \quad n = 1 \dots, \infty, \quad (10)$$

and  $\eta_{mn}^{(i)}$  being calculated using the dispersion equation

$$\eta_{mn}^{(i)} = \sqrt{\kappa_m^2 + \lambda_n^2 + \frac{s}{a_i}}. \quad (11)$$

224 Note that the development coefficients  $C_{mn}^{(i)}$  and  $D_{mn}^{(i)}$   
 225 standing for the upwards and downwards evanescent modes  
 226 have been normalised with respect to the values of the cor-  
 227 responding exponential terms at the lower  $z_i$  and upper  
 228  $z_{i-1}$  interface, respectively. There are different ways to  
 229 normalise the modes. The one chosen here assures that  
 230 all the corresponding exponents are negative, resulting in 246  
 231 values lower than one, and thus overflows are avoided. 247

In the air regions above and underneath the piece, only the terms vanishing to infinity must be kept, reducing the general expansion to the following expressions

$$T^{(0)} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} D_{mn}^{(1)} e^{-\eta_{mn}^{(0)} z} w_{mn}(x, y) \quad (12)$$

and

$$T^{(N+1)} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn}^{(N+1)} e^{\eta_{mn}^{(0)} (z - z_N)} w_{mn}(x, y) \quad (13)$$

232 for the regions above and below the piece, respectively,  
 233 with  $\eta_{mn}^{(0)} = \sqrt{\kappa_m^2 + \lambda_n^2 + s/a_0}$ ,  $a_0$  standing for the air  
 234 diffusivity. 248

235 The expansion coefficients in (7) and (12),(13) are de-  
 236 termined according to the standard procedure by applying 249  
 237 the continuity of the temperature and the normal to the  
 238 interface heat flux across the piece interfaces. 250

239 We treat the problem in two steps. First, the solution  
 240 for the flawless medium is developed, assuming a perfect  
 241 thermal contact between all layers. The effect of delam-  
 242 ination effects is accounted for in a second step by mod-  
 243 ification of the temperature continuity at the respective  
 244 interfaces.

### 2.3. Equation system for the flawless medium

245 Considering the interface between the  $i$  and the  $(i+1)$ th  
 layers, these two relations are written as

$$\left[ T^{(i)} - T^{(i+1)} \right]_{z=z_i} = 0 \quad (14)$$

and

$$\left[ J_n^{(i)} - J_n^{(i+1)} \right]_{z=z_i} = J_e, \quad (15)$$

respectively, with the normal heat flux in the  $i$ th layer being given by the Fourier's law

$$J_n^{(i)}(x, y) = -\kappa_i \frac{\partial T^{(i)}}{\partial z}. \quad (16)$$

$J_e$  stands for the incoming flux of the heat source, which for the class of problems which we are dealing admits a non-zero value only at the upper interface, where the thermal excitation is applied. Assuming an impinging beam of constant intensity  $J_e$  and of arbitrary support  $S(x, y)$ , the source term can be developed in terms of the modal functions  $w_{mn}(x, y)$  as

$$J_e(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn}^{(e)} w_{mn}(x, y), \quad (17)$$

where the development coefficients are given by the integrals

$$C_{mn}^{(e)} = J_e \int_0^{L_x} \int_0^{L_y} S(x, y) w_{mn}(x, y) dx dy. \quad (18)$$

The above integrals admit closed form expressions for the most usual shapes such as rectangular and circular.

Substituting (7),(12),(13) and (17) in (14),(15) and weighting with  $w_{mn}$  we obtain

$$\begin{aligned} C_{mn}^{(i)} e^{-\eta_{mn}^{(i)} d_i} + D_{mn}^{(i)} \\ - C_{mn}^{(i+1)} - D_{mn}^{(i+1)} e^{-\eta_{mn}^{(i+1)} d_{i+1}} = 0 \end{aligned} \quad (19)$$

and

$$\begin{aligned} -\eta_{mn}^{(i)} \kappa_i \left( C_{mn}^{(i)} e^{-\eta_{mn}^{(i)} d_i} - D_{mn}^{(i)} \right) \\ \eta_{mn}^{(i+1)} \kappa_{i+1} \left( C_{mn}^{(i+1)} - D_{mn}^{(i+1)} e^{-\eta_{mn}^{(i+1)} d_{i+1}} \right) \\ = C_{mn}^{(e)} \delta_{i,0}, \end{aligned} \quad (20)$$

where  $d_i = z_{i-1} - z_i$  and  $\delta_{i,0}$  is the Kronecker delta. Notice that for the first and last interface,  $C_{mn}^{(1)}$  and  $D_{mn}^{(N+1)}$  vanish.

Since the development basis is infinite, we need to truncate the sums (7),(12) and (13) in order to arrive at a solvable system of equations. We are allowed to do that since the series terms are decreasing for increasing mode numbers  $m$  and  $n$ . Thus, assuming an exact arithmetic, the error can be made arbitrarily small by judicious choice of the number of modes taken into account. Let  $N_m$  and  $N_n$  be this number for the modes along the  $x$  and  $y$  directions. The previous result for the continuity relations at the  $i$ th interface can be conveniently represented in a matrix form as follows

$$\mathbf{A}^{(i)} \mathbf{X}^{(i)} = \mathbf{B}^{(i)}, \quad (21)$$

with

$$\mathbf{A}^{(i)} = \begin{bmatrix} e^{-\eta^{(i)} d_i} & \mathbf{1} & -\mathbf{1} & -e^{-\eta^{(i+1)} d_{i+1}} \\ -\boldsymbol{\beta}^i e^{-\eta^{(i)} d_i} & \boldsymbol{\beta}^i & \boldsymbol{\beta}^{i+1} & -\boldsymbol{\beta}^{i+1} e^{-\eta^{(i+1)} d_{i+1}} \end{bmatrix} \quad (22)$$

$$\mathbf{X}^{(i)} = \left[ \mathbf{C}^{(i)} \quad \mathbf{D}^{(i)} \quad \mathbf{C}^{(i+1)} \quad \mathbf{D}^{(i+1)} \right]^T \quad (23)$$

and

$$\mathbf{B}^{(i)} = \begin{bmatrix} \mathbf{0} & \delta_{i,0} \mathbf{C}^{(e)} \end{bmatrix}^T. \quad (24)$$

251 The bold symbols in the system matrix (22) are under-  
 252 stood as diagonal matrices, namely  $\boldsymbol{\eta}^{(i)} = \text{diag}[\eta_{mn}^{(i)}]$ ,  
 253  $m = 0, \dots, N_m$ ,  $n = 0, \dots, N_n$  and  $\boldsymbol{\beta}^{(i)} = \kappa_i \boldsymbol{\eta}^{(i)}$ , and  
 254 the coefficient vectors such as  $\mathbf{C}^{(e)}$  are column vectors,  
 255 i.e.,  $\mathbf{C}^{(i)} = [C_{mn}^{(i)}]^{(i)}$ ,  $m = 0, \dots, N_m$ ,  $n = 0, \dots, N_n$ , etc.

256 Assembling the relations (21) for the  $N + 1$  interfaces  
 257 of the medium, we arrive at a linear system of equations,  
 258 which relates the  $2(N + 1)N_m N_n$  sought development coef-  
 259 ficients, i.e., the system rank is  $2(N + 1)N_m N_n$ . It should  
 260 be noticed here that the inversion of (21) using algorithms  
 261 for sparse matrices is equivalent to the explicit iterative  
 262 calculation of (7), (12),(13) or the reflection/transmission  
 263 coefficients approach adopted elsewhere, both from theo-  
 264 retical and computational point of view. The matrix form  
 265 representation adopted in this work allows a more compact  
 266 notation, which is the reason for choosing it.

#### 267 2.4. Introduction of delamination defects

268 Let us consider now the multilayer medium of the pre-  
 269 vious paragraph but this time we assume an imperfect con-  
 270 tact between the  $i$ th and the  $(i + 1)$ th layer, which can be  
 271 attributed to the existence of a thin void inclusion between  
 272 the two layers. This kind of imperfect contacts between  
 273 two adjacent layers is practically met in media affected by  
 274 delamination defects. The effect of the defect on the heat  
 275 propagation inside the medium can be taken into account  
 276 by locally modifying the continuity relation between the  
 277 two layers  $i$  and  $i + 1$ .

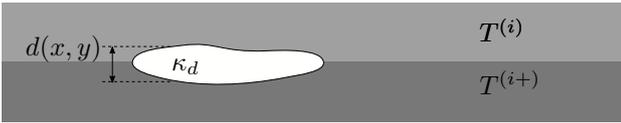


Figure 2: Void inclusion between two adjacent layers. The void is assumed homogeneous and much thinner than the embedding layers.

Consider the general case of a thin void inclusion depicted in Fig. 2, where  $\mathcal{D}$  is void domain and  $d(x, y)$  stands for its thickness, which in the general case is a function of the  $x, y$  position. By applying the Fourier's law across the void volume, one has

$$J_z = -\kappa_d \nabla T, \quad (25)$$

which for relatively small values of  $d(x, z)$  reduces to the finite difference relation

$$J_z(x, y) \approx -\kappa_d \frac{T^{(i)}(x, y) - T^{(i+1)}(x, y)}{d(x, y)}. \quad (26)$$

By introducing the local thermal resistance as [28]

$$R(x, y) = \frac{d(x, y)}{\kappa_d}, \quad (27)$$

the previous relation can be written as follows:

$$T^{(i)} - T^{(i+1)} = -R(x, y) J_z. \quad (28)$$

Substitution of (28) into (19) and taking (16) into account yields

$$\begin{aligned} & \sum_{m'=0}^{N_m} \sum_{n'=0}^{N_n} \left( \delta_{nm, n'm'} - R_{m'n'} \kappa_i \eta_{m'n'}^{(i)} \right) C_{m'n'}^{(i)} e^{-\eta_{m'n'}^{(i)} d_i} \\ & + \sum_{m'=0}^{N_m} \sum_{n'=0}^{N_n} \left( \delta_{nm, n'm'} + R_{m'n'} \kappa_i \eta_{m'n'}^{(i)} \right) D_{m'n'}^{(i)} e^{-\eta_{m'n'}^{(i)} d_i} \\ & - C_{mn}^{(i+1)} - D_{mn}^{(i+1)} e^{-\eta_{mn}^{(i+1)} d_{i+1}} = 0, \end{aligned} \quad (29)$$

where  $R_{mn}$  stands for the weighted thermal resistance function

$$R_{mn} = \int_{-L_x}^{L_x} \int_{-L_y}^{L_y} R(x, y) w(x, y) dx dy. \quad (30)$$

Adopting again the matrix notation presented above, the continuity relations for the  $i$ th layer become

$$\left( \mathbf{A}^{(i)} + \delta \mathbf{A}^{(i)} \right) \mathbf{X}^{(i)} = \mathbf{B}^{(i)} \quad (31)$$

with  $\mathbf{A}^{(i)}$ ,  $\mathbf{X}^{(i)}$  and  $\mathbf{B}^{(i)}$  being given by (22), (23) and (24) respectively, and

$$\delta \mathbf{A}^{(i)} = \mathbf{R} \begin{bmatrix} -\boldsymbol{\beta}^i & \boldsymbol{\beta}^i & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad (32)$$

where this time  $\mathbf{R}$  is a full matrix that couples all modes together. The introduction of the defect, in other words, comes at the cost of the partial loss of the system sparsity. An interesting limiting case, though, is the one where the support of the delamination defect exceeds the computational domain, i.e.  $R(x, y) = c$ , for  $-L_x \leq x \leq L_x$  and  $-L_y \leq y \leq L_y$ , with  $c$  constant. In this case  $\mathbf{R}$  becomes diagonal and the sparsity of the original system is restored. This is the quasi-1D case in the sense that the considered geometry is invariant along  $x$  and  $y$  directions, yet excited via an arbitrary flux profile. An intermediate case would be also having an infinite source along one of the  $x$  or  $y$  directions. In this latter case, the  $\mathbf{R}$  matrix becomes block-diagonal, i.e., it has the form of a Kronecker product of a diagonal and a full submatrix.

#### 293 2.5. Inverse Laplace transform

For the development of the semi-analytical solution we have worked so far entirely in the Laplace domain. We are interested though in the transient response of a number of observables such as the temperature evolution at specific points of the geometry and especially on the two surfaces of the medium, which is actually the only accessible experimental information of the method. In order to obtain the temporal evolution of the temperature field the solution in

the Laplace domain should be transformed back into the time domain.

There are several methods reported in the literature for performing this inversion. In this work we have chosen to carry out the inversion numerically using the Stehfest's algorithm [39, 40], which is an improved variant of Gaver's method [41]. This method is well adapted to the problem of impulse thermography studied here [28].

If  $F(s)$  is the known Laplace transform of the function  $f(t)$ , evaluated at  $s = a_j/t$  where  $a_j = j \ln(2)$ , then an approximate value of this function at time  $t$  can be calculated as

$$f(t) \cong \frac{\ln(2)}{t} \sum_{j=1}^N V_j F\left(j \frac{\ln(2)}{t}\right). \quad (33)$$

The coefficients  $V_j$  are given by the following expression for an even value of  $N$ :

$$V_j = (-1)^{j+N/2} \times \sum_{k=\text{Int}((j+1)/2)}^{\text{Min}(N/2, j)} \frac{k^{N/2(2k)!}}{(N/2 - k)!k!(k-1)!(j-k)!(2k-j)!}. \quad (34)$$

In this equation 'Int' designates the integer part of a real number and 'Min' the minimum of two numbers.

Applying (34) at a number of preselected instances  $t$ , we then obtain the temporal profile of our observable at those points. The optimal choice of the points, itself, will depend on the thermal signal processing that we wish to apply. In the following, the TSR approach is used for the representation of the thermograms at the plate interfaces, and consequently the choice of the suitable sampling points will be based on this approach.

### 3. Defect shape reconstruction from the thermal images

In order to properly characterize subsurface defects in laminates, the signals recorded by an infrared camera should be processed by some algorithms. Data processing algorithms in TNDT are either one-dimensional or two-dimensional, with the first ones being applied to pixel-based temperature evolution in time and the second ones applied to single images. The time series of each recorded pixel is separately evaluated, usually to find any alterations of material properties or the depth of those alterations. Single IR images are normally filtered or segmented to reduce random noise or to analyse geometrical features of the areas of interest.

Since its introduction [42], the thermographic signal reconstruction (TSR) method has emerged as one of the most widely used methods for enhancement, analysis and compression of raw thermographic sequences. The technique was originally developed for pulse thermography to improve contrast results and is one of the pixel-based methods.

One of the well-established edge-detection algorithms is the so-called Canny algorithm [24]. The method processes an input image and detects the edges of any present objects (defects). Because of a better signal to noise ratio and detection accuracy, the Canny operator becomes the evaluation criterion of other methods. The algorithm consists of Gauss filtering, gradient calculation, non-maximum suppression, double thresholding, checking the edges and connecting the edges.

In this section, the theoretical background of the two mentioned methods is given and numerical results from their application for the detection of delamination-type defect in planar media shown.

#### 3.1. Thermographic signal reconstruction

The technique is based on the pixel-based evaluation of the time series which represent the surface temperature variation over time. The technique consists of two basic steps. The first step is the fitting of the recorded time series in the log-log space by a logarithmic polynomial of degree  $n$ . This step provides a significant compression of the raw data. In a later step, the reconstruction of the temperature signals in the logarithmic domain using the polynomial has to be performed, providing noise-reduced copy of each pixel time series. Fitting to the thermograms highly depends on the time window chosen and the polynomial degree. Thus, the choice of the time window has to be made with the objective to consider only the part of the thermograms influenced by the physical phenomena to characterize, and the polynomial degree has to be defined considering that this will be a trade-off between accuracy of signal reproduction and denoising.

Especially, in a defect-free sample, the thermal diffusion in the lateral direction could be neglected compared to the normal direction, so one can consider the one-dimensional heat equation for the thermography. In principle, the method exploits the well-known observation that in a semi-infinite flawless sample, or in a very thick slab, the surface temperature response to instantaneous uniform heating, ideal impulsive heat flux, is described by the one-dimensional heat diffusion equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2}, \quad (35)$$

where  $\alpha = \frac{\kappa}{\rho c}$  is the thermal diffusivity of the material with the solution

$$T(z, t) = \frac{Q}{\epsilon \sqrt{\pi t}} e^{-\frac{z^2}{4\alpha t}} \quad (36)$$

at distance  $z$  from its surface, where  $\epsilon = \sqrt{\kappa \rho c}$  is the heat effusivity and  $Q$  is the quantity of energy absorbed by the surface. Since thermal imaging is only applicable to surface temperature, from Eq. (36) one has the temperature increase  $\Delta T$  (thermogram) as a function of time  $t$  at the surface:

$$\Delta T(t) = \frac{Q}{\epsilon \sqrt{\pi t}}. \quad (37)$$

372 The one-dimensional approximation of Eq. (37) assumes<sup>404</sup>  
 373 that the lateral diffusion components more or less cancel<sup>405</sup>  
 374 in a defect-free sample. However, in the presence of an  
 375 adiabatic subsurface boundary such as a void, a wall, a  
 376 buried defect, or in the case of a multi-layer sample the incident heat flow from the sample surface is impeded, and this solution no longer applies locally. The effect of a wall is shown in the left plot in Fig. 3 where the surface temperature for a semi-infinite sample is compared with the case of the presence of an adiabatic wall at three different depths. The effect of a buried defect in the thermograms is shown in the right plot in Fig. 3 where the separation of the temperature response at the surface of a solid between a sound area and a defected area should be simple when these defects are large or very close to the surface. However, when one attempts to detect thin buried defects, the effects of infrared camera noise as well as the complexity found in many samples limit the ability to discriminate between sound and flawed areas.

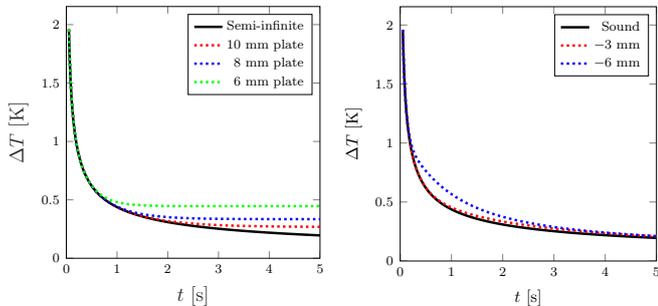


Figure 3: Surface temperature decay curves for a single point of a<sup>407</sup>  
 steel plate. *Left*: Comparison of temperature time plot of a semi-<sup>408</sup>  
 infinite sample with three adiabatic samples of different thickness.  
*Right*: Comparison of temperature time plot of a sound sample with  
 two defected samples at different depth.

Considering the time evolution of the surface temperature in the logarithmic domain, where Eq. (37) can be written in the logarithmic scale, additional insight into the surface temperature response during the cooling period of the sample is gained:

$$\ln(\Delta T) = \ln\left(\frac{Q}{\epsilon\sqrt{\pi}}\right) - \frac{1}{2}\ln(t). \quad (38)$$

391 In Eq (38) the time dependence has been separated from  
 392 the input energy and material properties. Only the offset  
 393 of the response will change as the sample material and the  
 394 input energy vary.

395 For a semi-infinite piece, Eq. (38) describes a straight  
 396 line with slope equal to  $-1/2$ , as pictured in Fig. 4. In the  
 397 case of an adiabatic plate, the response deviates from the  
 398 straight line at a particular time, depicted in the left plot in<sup>409</sup>  
 399 Fig. 4. This particular time is correlated with the thickness<sup>410</sup>  
 400 of the plate. In the presence of a subsurface defect in a<sup>411</sup>  
 401 plate, or in a semi-infinite solid, the time evolution plot of<sup>412</sup>  
 402 the temperature corresponding to those pixels depart from<sup>413</sup>  
 403 that behaviour in a particular time but in a different way<sup>414</sup>

comparing to the case of the adiabatic flaw-free plate, as  
 shown in the right hand plot in Fig. 4.

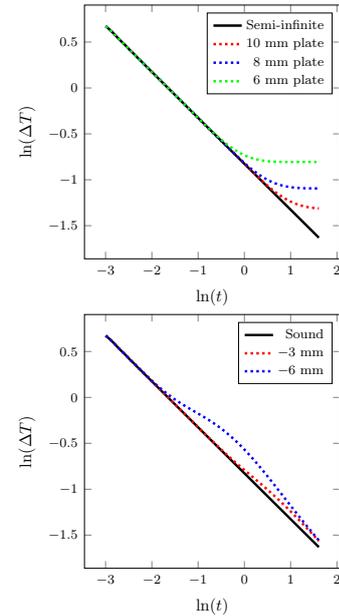


Figure 4: Surface temperature decay curves for a single point of a  
 steel plate in the logarithmic scale. *Left*: Comparison of temperature  
 time plot of a semi-infinite sample with three adiabatic samples of  
 different thickness in the logarithmic space. *Right*: Comparison of  
 temperature time plot of a sound sample with two defected samples  
 at different depth in the logarithmic space.

In Fig. 4 the thermograms have been normalised based  
 on Eq. (38) where the term which describes the source and  
 the material effect have been removed.

For a given pixel, the response given by Eq. (38) can  
 be approximated, in general, by a function or set of or-  
 thogonal functions. The TSR method uses a polynomial  
 series to fit the experimental data in log-log space:

$$\ln(\Delta T) = \sum_{n=0}^N a_n [\ln(t)]^n. \quad (39)$$

The fitting of the log-log thermogram, for each recorded  
 pixel  $(i, j)$ , by the logarithmic polynomial replaces the full  
 sequence of the temperature response images  $T(i, j, t)$  by  
 the series of  $(n + 1)$  images of the polynomial coefficients  
 $a_0(i, j), \dots, a_n(i, j)$ . Once the time evolution of each pixel  
 has been approximated by Eq. (39), the original data can  
 be reconstructed as

$$\Delta T = \exp\left(\sum_{n=0}^N a_n [\ln(t)]^n\right). \quad (40)$$

Thus, it is only necessary to store the polynomial coef-  
 ficients  $a_n$  and reconstruct the images stack at required  
 time samples. This approach provides a significant degree  
 of data compression.

A low-degree polynomial expansion is applied usually  
 to serve as a low-pass filter. Thus, this approximation pre-

415 serves the essential thermal response, while rejecting non-  
 416 thermal noise contributions. The use of higher-order poly-  
 417 nomials reproduces the original data and replicates also  
 418 part of the noise that appears in the later, low-amplitude  
 419 data. In Fig. 5 reconstructed noisy signals by using poly-  
 420 nomial degrees  $n = 7$  and  $17$ , with and without the pres-  
 421 ence of a defect, are compared, and the trade-off between  
 422 accuracy of signal reproduction and signal denoising is  
 423 clearly exhibited.

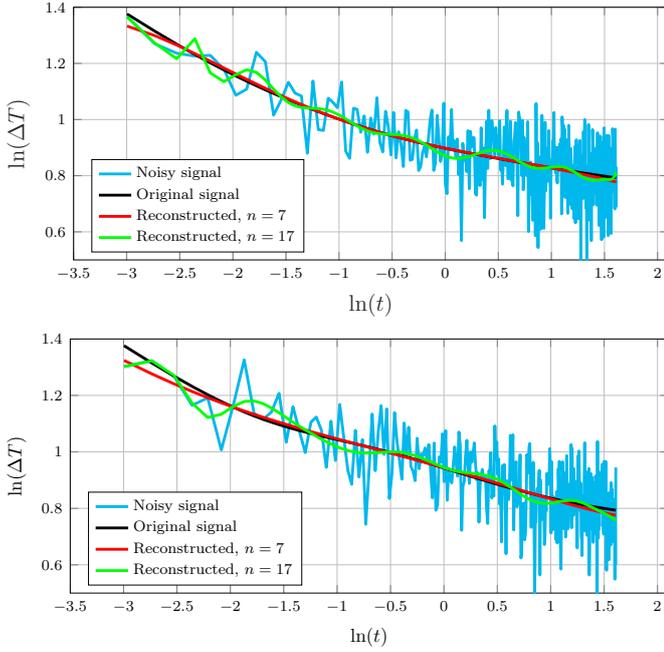


Figure 5: Regression by a logarithmic polynomial of degree  $n$  equal to 7 and to 17, for a sound area, in the top plot, and a flawed area, in the bottom plot.

424 The use of TSR effectively removes the temporal noise  
 425 from the recorded raw signals. However, the reduction  
 426 of temporal noise does not necessarily increase the flaw  
 427 detectability. This can be partially addressed by the com-  
 428 putation of the first and second time derivatives, using  
 429 Eq. (39), without additional noise contributions, which  
 430 leads to equations

$$431 \quad \frac{d \ln(\Delta T)}{d [\ln(t)]} = \sum_{n=1}^N n a_n [\ln(t)]^{n-1}, \quad (41)$$

$$432 \quad \frac{d^2 \ln(\Delta T)}{d [\ln(t)]^2} = \sum_{n=2}^N n(n-1) a_n [\ln(t)]^{n-2}, \quad (42)$$

433 for the first and the second derivative, respectively. The  
 434 temperature in the logarithmic scale is compared with the  
 435 first and the second time derivative for a sound area and  
 436 a flawed area in Fig. 6.

437 The denoising provided by TSR is not limited to the  
 438 time signals but has been partially transmitted to the spa-  
 439 tial signals also, this is an indirect effect of the method.  
 440 The derivatives, by definition, are much more sensitive to

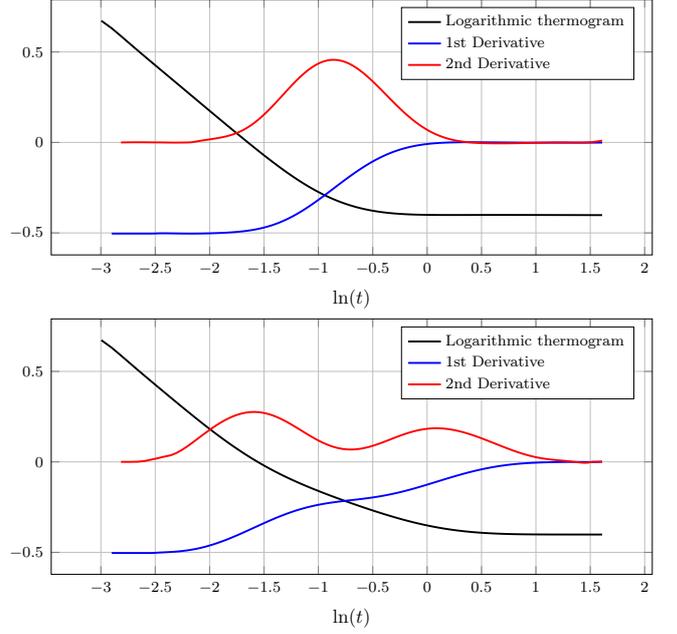


Figure 6: Plots of the logarithmic thermogram, 1st and 2nd logarithmic time derivatives for a thick steel plate. *Top*: Homogeneous sound plate. *Bottom*: Homogeneous plate with an embedded flaw.

441 small changes in amplitude than the raw signal. However,  
 442 after the application of the TSR that acts as a low-pass  
 443 filter, the derivatives are less sensitive to random signal  
 444 fluctuations. In Fig. 7 the raw signals versus the hori-  
 445 zontal position plot of the steel defect sample and the TSR  
 446 first derivative are compared.

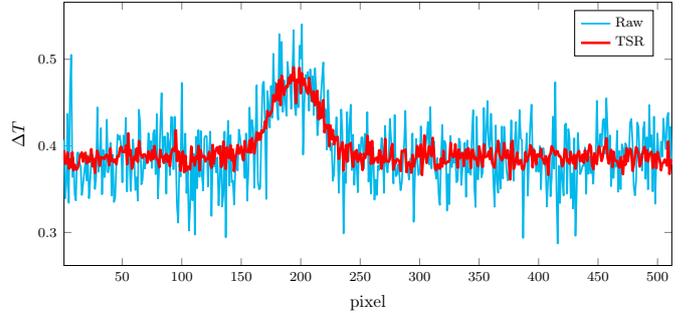


Figure 7: Comparison of raw and TSR results for a horizontal line through the centre row of delamination.

The derivative signals have been partially denoised by TSR also. The high sensitivity of the derivatives to small changes makes them ideal for the detection of features that can be undetectable in the original data. Comparing now images obtained from TSR, first and second derivatives, in Fig. 8, it is obvious that the images are sharper and the detectability of small flaws has been significantly improved.

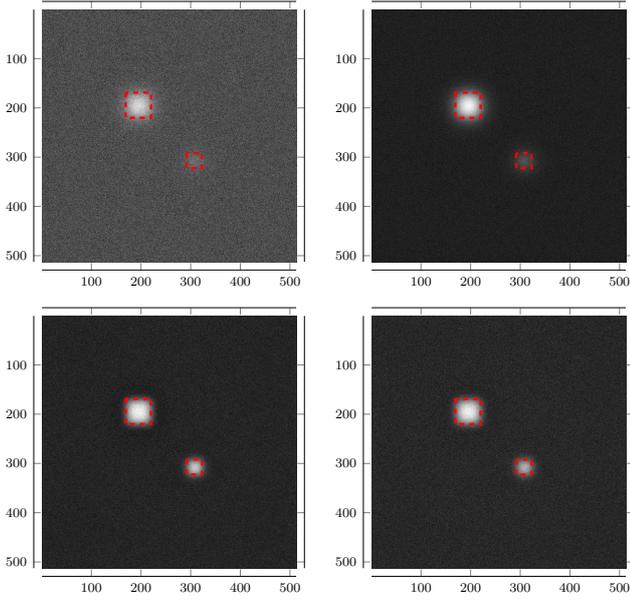


Figure 8: Comparison of raw image in the left top corner, TSR image in the right top corner, 1st derivative image in the left bottom corner and 2nd derivative image in the right bottom corner.

### 3.2. Shape reconstruction using the Canny edge detection algorithm

In its classical form, the Canny algorithm, in its first step, processes the image smoothly through Gaussian convolution and obtains the gradient image through differential operation on the image which is processed via Gaussian convolution. Consider the two-dimensional Gaussian function

$$G(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right), \quad (43)$$

with mean  $\mu = 0$  and standard deviation  $\sigma$ . The parameter  $\sigma$  is the width of the Gaussian filter and directly determines the effect of filtering where larger  $\sigma$  provides smoother images. The filtered image is derived from the convolution

$$\widehat{\mathcal{I}}(x, y) = G(x, y) * \mathcal{I}(x, y), \quad (44)$$

where  $\mathcal{I}(x, y)$  is the original image matrix. Making use of Gaussian function's separability,  $\nabla G$  can be decomposed into two one-dimension filters:

$$\frac{\partial G}{\partial x} = kx \exp\left(-\frac{x^2}{2\sigma^2}\right) \exp\left(-\frac{y^2}{2\sigma^2}\right), \quad (45a)$$

$$\frac{\partial G}{\partial y} = ky \exp\left(-\frac{y^2}{2\sigma^2}\right) \exp\left(-\frac{x^2}{2\sigma^2}\right). \quad (45b)$$

By convolving these equations with the image we obtain:

$$L_x = \frac{\partial G}{\partial x} * \mathcal{I}(x, y), \quad (46a)$$

$$L_y = \frac{\partial G}{\partial y} * \mathcal{I}(x, y), \quad (46b)$$

where in a matrix representation this can be written as:

$$L_x = K_x * \mathcal{I}(i, j), \quad (47a)$$

$$L_y = K_y * \mathcal{I}(i, j), \quad (47b)$$

where  $i$  and  $j$  are the coordinates of a pixel in the image.

This is a way to compute the gradient, but not the only one. Different kernels can be used to calculate the image gradient. This could separate the denoising part of the algorithm from the computation of the gradient. For the traditional Canny algorithm, two  $2 \times 2$  convolution operators  $K_x$  and  $K_y$  are deployed to calculate the image gradient in the  $x$  and  $y$  directions, respectively. These operators are written as:

$$K_x = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}, \quad (48a)$$

$$K_y = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}. \quad (48b)$$

In general,  $2 \times 2$  or  $3 \times 3$  kernels based on finite differences are used for the computation of the gradient. In the literature one can find many other derivative kernels which provide also some smoothing in the data, with the most used ones being the Sobel operator:

$$K_x = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix}, \quad (49a)$$

$$K_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ -1 & -2 & -1 \end{bmatrix}. \quad (49b)$$

After convolving the image with the kernels, the gradient-component intensity of the image is derived from

$$M(i, j) = \sqrt{L_x^2(i, j) + L_y^2(i, j)}, \quad (50)$$

and its normal vector direction at the pixel  $(i, j)$  is defined as

$$\theta(i, j) = \arctan\left[\frac{L_x(i, j)}{L_y(i, j)}\right]. \quad (51)$$

To follow, after smoothing the image using Gaussian smoothing and convolving it with derivative kernels, one ends up with the gradient magnitude image  $M(i, j)$  which reflects the edge intensity at the pixel  $(i, j)$ , and  $\theta(i, j)$ , which reflects the normal vector at the pixel  $(i, j)$  in the image. Edges of objects can be extracted from the gradient component intensity image, but they will be quite blurry. In this step, the algorithm aims at thinning those edges by setting the pixels around local maxima in the gradient image  $M$  to 0.

Firstly, the direction angle is rounded to  $0^\circ$ ,  $45^\circ$ ,  $90^\circ$ ,  $135^\circ$  for the relative position in adjacent pixels of the image. Aiming at every pixel whose value is non-zero, the gradient-component intensity of a candidate pixel  $M(i, j)$  is compared with two adjacent pixels along the rounded direction angle. The candidate pixel is preserved only if

its gradient component intensity is the largest. Otherwise, it is set to zero. Let the processed image be  $\widehat{M}(i, j)$ .

The non-maximum suppression gives the non-zero pixels providing more accurate approximation regarding the edges of the objects in the processed image  $M(i, j)$ . These pixels are taken as the edge pixels. Due to noise in the original image,  $\widehat{M}(i, j)$  contains pixels depicting false edges, the spurious edge response. To mitigate these spurious edges, hysteresis tracking is performed using dual thresholding by setting a high  $\tau_h$  and a low  $\tau_l$  threshold parameter. Edge pixels which have gradient larger than  $\tau_h$  are added automatically to the final binary image and are considered as strong edge pixels. In opposition, edge pixels with a gradient lower than  $\tau_l$  are considered as phantom edges and are discarded. The remaining pixels with a gradient value between  $\tau_l$  and  $\tau_h$  are considered as weak edges and are added to the final binary image only if they are connected with a strong edge pixel. When none of the 8-connected neighbourhood pixels is a strong pixel, the candidate pixel is suppressed.

The choice of thresholds is very crucial for the success of the method. The algorithm can wipe off most of the spurious edges while increasing the value of  $\tau_h$ , but meanwhile, some edges may be missed. On the other hand, by decreasing the value of  $\tau_l$  more information about the edges will be preserved but the edge's characteristic will become less and less at the point where the true edges will be missed. Auto-select thresholding value is a difficult task. At present, there are many kinds of methods in selecting threshold values. The more widely used is the Otsu method [43] but also other methods based on histogram, maximum entropy, or statics are used [44, 45, 46].

The Otsu method has the best threshold value in the statistical sense and is the most stable method in the image threshold segmentation. The method has been used here to choose the value of  $\tau_h$  automatically. The method assumes that the pixels of the image to be thresholded can be separated into two classes, *e.g.* foreground and background, then calculates the optimum threshold separating those two classes so that their combined spread is minimal.

Suppose that  $G = [0, L - 1]$  is the range of greyscale image  $F$  and  $P_i$  is the probability of every greyscale, the threshold value  $\tau$  has splitted the image in two classes which are  $C_0 = [0, \tau]$  and  $C_1 = [\tau + 1, L - 1]$ . The two classes probabilities are

$$\alpha_0 = \sum_{i=0}^{\tau} P_i$$

and

$$\alpha_1 = 1 - \alpha_0,$$

respectively. The average grey values of the two classes are

$$\mu_0 = \sum_{i=0}^{\tau} \frac{iP_i}{\alpha_0} = \frac{\mu_{\tau}}{\alpha_0}$$

and

$$\mu_1 = \sum_{i=\tau+1}^{L-1} \frac{iP_i}{\alpha_1} = \frac{\mu - \mu_{\tau}}{1 - \alpha_0},$$

respectively, therein

$$\mu = \sum_{i=0}^{L-1} iP_i, \quad \mu_{\tau} = \sum_{i=0}^{\tau} iP_i.$$

The criterion function has been defined as variance between the two classes, expressed as

$$\eta^2(\tau) = \alpha_0(\mu_0 - \mu)^2 + \alpha_1(\mu_1 - \mu)^2 \quad (52)$$

$$= \alpha_0\alpha_1(\mu_0 - \mu_1)^2. \quad (53)$$

The optimal threshold value  $\tau^*$  is given by

$$\eta^2(\tau^*) = \max_{0 \leq \tau \leq L} \eta^2(\tau). \quad (54)$$

Threshold  $\tau^*$  will be used as the high threshold parameter  $\tau_h$ . The value of the low threshold  $\tau_l$ , usually, is set to be  $\tau_l = \frac{\tau_h}{2}$ .

The last step of the algorithm is the connection of already detected edges on the binary image under some restrictions. These restrictions are criteria set upon the gradient value  $\widehat{M}(i, j)$  and the gradient angle  $\theta(i, j)$  of the non-edge pixels between two edges. If the gradient is larger than a given value and the gradient angle is close to zero, the pixels between the two edges are added to the binary image.

The output of the Canny algorithm is a binary image which contains the edges of any object present in the initial image.

## 4. Results

### 4.1. Description of the configuration

In this section, the proposed multi-step flaw characterization technique will be demonstrated by employing the forward model developed in Sec. 2, and the image processing techniques described in Sec. 3. Temperature signals, produced by the three-dimensional TREE model, have been corrupted with Gaussian noise of different levels to simulate temperature signals collected by an infrared camera and from now on are called raw signals. Other types of noise beyond Gaussian could be used in this part but this is out of our scope. The detection and characterization procedure has been divided into two parts. The first step concerns the detection and shape reconstruction of candidate flaws and the second step, their characterization through an iterative parameter estimation technique regularised by the information gained from the first step.

Concerning this section, the three-dimensional model developed in Sec. 2 has been used to compute the temperature field, for the general configuration depicted in Fig. 9.

These signals are supposed to be collected at the front, or upper surface of the work piece. A grade 4340 steel plate

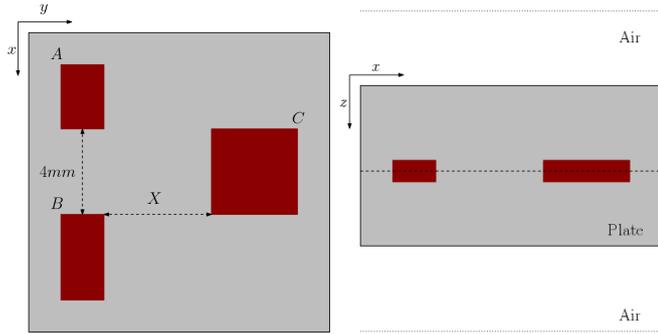


Figure 9: Sketch of the configuration in the  $(x, y)$  and  $(x, z)$ -plane, left and right respectively.

with thermal conductivity  $k = 44.5$  W/mK, heat capacity  $C_p = 475$  J/kgK, density  $\rho = 7850$  kg/m<sup>3</sup> and thickness  $d = 3$  mm is used. The plate is considered to be infinite in the  $x$ - and  $y$ -direction where the area of interest is a  $40 \times 40$  mm<sup>2</sup> rectangle. Three well-defined air-filled defects, named  $A$ ,  $B$  and  $C$ , which simulate delaminations of different thickness,  $d_A = 3 \times 10^{-3}$  mm,  $d_B = 2 \times 10^{-3}$  mm,  $d_C = 1 \times 10^{-3}$  mm and of different sizes  $3 \times 2$  mm<sup>2</sup>,  $4 \times 2$  mm<sup>2</sup>,  $4 \times 4$  mm<sup>2</sup>, respectively, have been embedded into the plate with their larger faces set to be parallel to the plate surface. As an excitation term, a flash lamp, set above the plate and parallel to its surface, depositing a heating power density of  $Q = 10^4$  W/m<sup>2</sup> at the surface of the plate, has been modelled as a Dirac's delta function in time, whereas its spatial distribution is considered to be uniform and covers all the domain of interest, as already done before.

#### 4.2. TSR and noise reduction

The described configuration is used in this part with the addition that the flaws are located in the middle of the plate along the  $z$ -axis.

The shape reconstruction part of the technique starts by applying the TSR method to the noisy signals. This provides the polynomial approximation of the signals, say, the matrix  $\mathcal{P}$  of dimensions  $N_x \times N_y \times (p+1)$ , where  $N_x, N_y$  are the pixels numbers along the  $x$ - and  $y$ -directions, and  $p$  is the polynomial degree. Polynomial coefficients of the first and second time derivatives are stored in the matrices  $\mathcal{P}_1$  and  $\mathcal{P}_2$ , respectively. The time-dependent temperature field in the logarithmic scale, as well as the first and second time derivatives, can be reconstructed using the polynomial matrices  $\mathcal{P}, \mathcal{P}_1, \mathcal{P}_2$  and stored in the matrices  $\mathcal{I}, \mathcal{I}_1, \mathcal{I}_2$ . The reconstruction of the time signals derives time frames which are smoother and suitable for defect detection. The reconstructed second time derivative matrix,  $\mathcal{I}_2$ , will be used for the detection of the time which corresponds to the best frame.

The choice of the polynomial degree is a very crucial task for the TSR technique, here we choose to work with 7<sup>th</sup> degree polynomials that were shown to approximate the original signal with high accuracy and filtering most of

the noise as observed from the log – log plots in Fig. 5. In Fig. 10 the reconstructed signals versus time are compared with the raw noisy signals as well as with the synthetic signals as a reference. The signals correspond to two different pixels with the first one being at the centre of the plate where it is considered to be a sound area and the second one at the centre of the largest defect, named  $C$ , left hand and right hand plot in Fig. 10, respectively. It is clear from Fig. 10 that most of the noise has been significantly filtered through the TSR method and the original signals have been reconstructed with great accuracy.

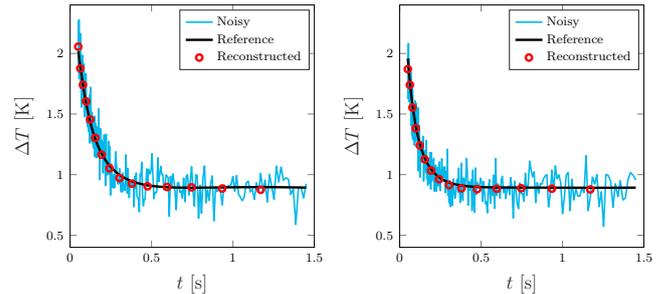


Figure 10: Reconstruction of temperature noisy signals with the TSR technique for a sound area and a flawed area. Comparison of the reconstructed signals with the reference signals and the noisy signals. *Left*: Time signals corresponding to a sound area of the plate. *Right*: Time signals corresponding to a flawed area of the plate.

An additional important detrimental factor for the raw signal is the finite camera readout precision, which is superposed to the additive signal noise. In order to test the algorithm robustness under more realistic conditions of finite camera readout, a minimum temperature resolution has been assumed, resulting in a rounded raw temperature signal with a precision of  $0.05^\circ\text{C}$ . The raw signal deformation when a finite precision is considered is shown in Fig. 7. In the same figure is also given the comparison of the resulting signal with infinite and finite temperature resolution after the application of the TSR. As can be seen from the comparison, the finite round-off error of the camera has a visible impact on the raw signal itself, yet the effect is relieved to a satisfactory degree when applying the TSR algorithm. Note that the two TSR signals (with zero and finite round-off error) have been obtained using the same polynomial degree.

#### 4.3. Shape reconstruction

The choice of the most suitable frame for shape reconstruction is crucial at this point. The best frame for the shape reconstruction algorithm should correspond to a time instant which maximises the contrast of the image. The contrast for defects of different thicknesses, located at different depths, reach their maximum contrast at different times [13]. To compute the contrast, one needs information about where the flaws are located and a reference sound area. Since no reference data will be used, so the computation of the contrast could be considered as

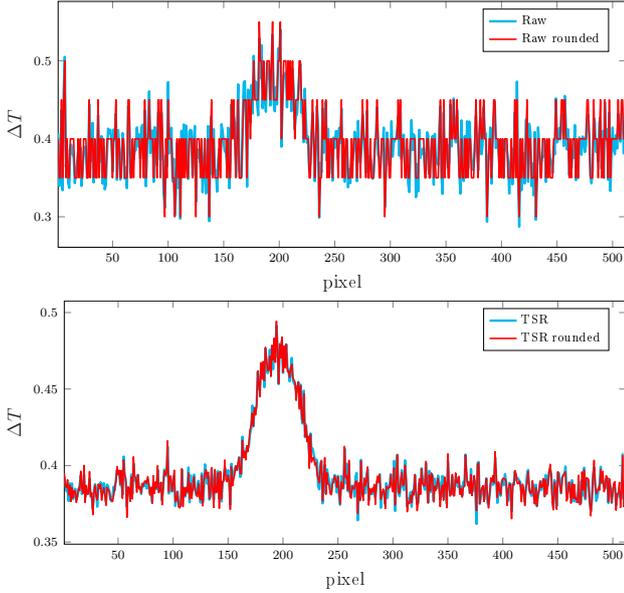


Figure 11: Comparison of the thermal signal obtained using zero and finite round-off error for a horizontal line through the centre row of delamination. *Up*: Raw signal. *Down*: Signal after application of the TSR algorithm. The TSR seems to remove the finite precision effect yielding an SNR comparable with the one obtained for zero round-off.

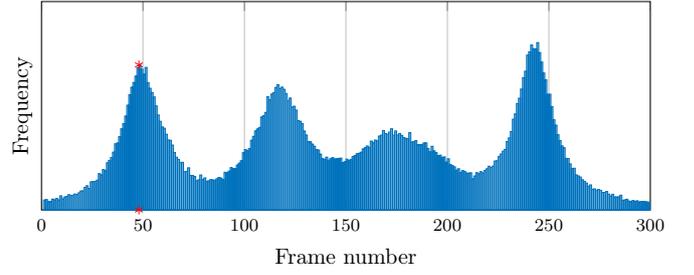


Figure 12: Frequency of the sign changes of the second time derivative for all pixels in each frame of the reconstructed signals.

693 understanding the relative depth of the flaws and their rela-  
 694 tive thickness since flaws with significantly different thick-  
 695 ness or located at different depth will appear with different  
 696 colours.

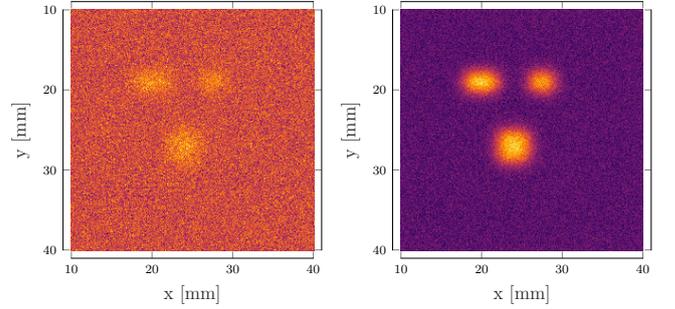


Figure 13: Comparison of a raw image with the reconstructed image corresponding to the same optimal frame number. *Left*: Raw image,  $\mathcal{I}$ . *Right*: Reconstructed image,  $\mathcal{I}^*$ .

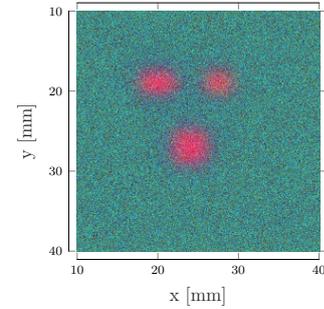


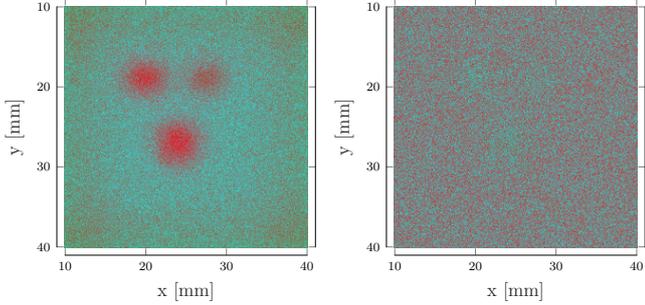
Figure 14:  $\text{RGB}(\mathcal{I}^*, \mathcal{I}_1^*, \mathcal{I}_2^*)$  image corresponding to the optimal frame number.

662 an option, the second time derivative matrix  $\mathcal{I}_2$  will be  
 663 used. A criterion upon the second time derivative for each  
 664 pixel will be set instead. Each variation that occurs in  
 665 the time-dependent signals, which is a result of the altera-  
 666 tion of material thermal properties at a specific depth, is  
 667 indicated by a change of the first time derivative of the sig-  
 668 nals. This change on the first time derivative of the signal  
 669 causes the second time derivative to change significantly.  
 670 The times at which the second time derivative reaches its  
 671 local maxima are correlated with the times where the abso-  
 672 lute contrast of the second derivative reaches its local  
 673 maxima. In order to have an early flaw identification, and  
 674 avoid the image blurring at later times, we take into ac-  
 675 count the times where the second time derivative changes  
 676 sign. For each frame, the frequency of the sign changes of  
 677 the second time derivative will be computed and the frame  
 678 with the maximum frequency chosen as shown in Fig. 12,  
 679 where the maximum frequency has been marked with a red  
 680 asterisk. If we have more than one candidate frame, the  
 681 frame that corresponds to the earlier time will be chosen.  
 682 This results in a sharper image but in a multilayer con-  
 683 figuration could result in loss of information about deeply  
 684 buried defects, i.e. very close to the bottom surface. In  
 685 such a case multiple images could be used.

686 The image corresponding to that frame number is shown  
 687 in Fig. 13, left, and been named  $\mathcal{I}^*$ . The effect of the tech-  
 688 nique on partially denoising the signals in space can be  
 689 seen by the comparison of the derived image with the raw  
 690 image corresponding to the same frame number, Fig. 13,  
 691 right. An RGB image constructed from the triplet  $\mathcal{I}^*,$   
 692  $\mathcal{I}_1^*, \mathcal{I}_2^*$  is shown in Fig. 14. The later is useful in un-

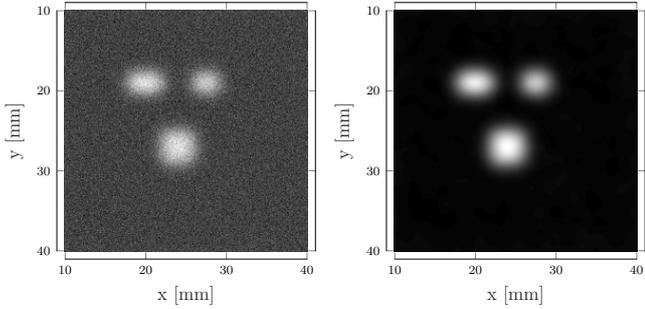
A similar RGB image can be constructed from the monomials but that representation is quite noisy when a high degree polynomial is used. The degree of the polynomials used here,  $p = 7$ , is considered to be high compared with the short time period of the recorded signals and the noise level. For a longer time period or lower noise level, an RGB image reconstructed using the monomials could be as useful as the image in Fig. 14. We illustrate this in Fig. 15, where signals with higher SNR (40 dB) are used to reconstruct RGB images using  $5^{th}$  and  $7^{th}$  degree polynomials. In Fig. 15, left, the monomials  $-5/5, 4/5, 3/5$  are used to

708 form the image and in Fig. 15, right, the used monomials  
 709 are the  $-7/7, 6/7, 5/7$ . It is clear from the figures that a  
 710 lower degree polynomial will offer more qualitative infor-  
 711 mation about the flaws that a higher degree polynomial  
 712 which inherits a larger part of noise.



735 Figure 15: Reconstructed RGB image from the projection of three  
 736 monomials after applying TSR on signals with SNR = 40 dB using  
 737 polynomials of 5<sup>th</sup> degree, *left*, and 7<sup>th</sup> degree, *right*.  
 738

713 At this point, the image  $\mathcal{I}^*$  will be provided to the  
 714 Canny algorithm for the detection and the reconstruction  
 715 of defects' shape. The first step of the algorithm con-  
 716 sists of a Gaussian smoothing of which the image will be a  
 717 subject. Thus the parameter  $\sigma$  that defines the amount  
 718 of smoothing will be provided to the Canny algorithm  
 719 since the smoothing algorithm has been integrated into  
 720 the Canny algorithm. In our case, we perform a slight  
 721 smoothing by using  $\sigma = 5$ . A comparison in greyscale of  
 722 the input image and the smoothed one is shown in Fig. 16,  
 723 where the two images have been rescaled.



749 Figure 16: The impact of the Gaussian smoothing of the input image  
 750 to the Canny algorithm shown in grey scale. *Left*: The noisy input  
 751 image  $\mathcal{I}^*$ . *Right*: The smoothed image  $\widehat{\mathcal{I}^*}$ .  
 752  
 753  
 754

724 A gradient magnitude image  $M$ , Fig. 17, left, of the  
 725 smoothed image  $\widehat{\mathcal{I}^*}$ , Fig. 16, right, will be derived after its  
 726 convolution with the derivative kernel, and in this case, the  
 727 Sobel kernel has been used. By applying non-maximum  
 728 suppression to the gradient image, all values along the line  
 729 of the gradient that are not peak values of the ridge have  
 730 been suppressed. This leads to the image  $\widehat{M}$ , Fig. 17, right,  
 731 which contains one-pixel- wide edges.

732 Due to noise in the original image,  $\widehat{M}$  contains pixels  
 733 depicting false edges and to mitigate these spurious edges,  
 734 hysteresis tracking is performed using dual thresholding.

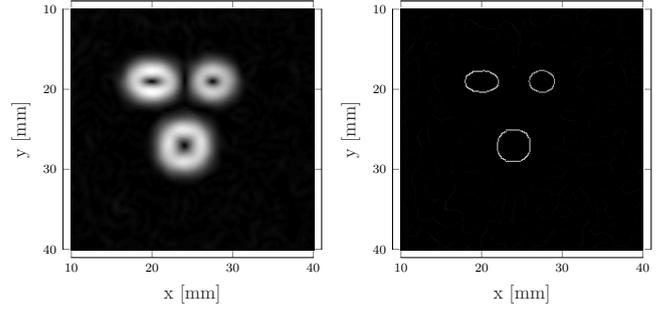


Figure 17: The impact of the non-maximum suppression on the  
 gradient intensity image. *Left*: The gradient intensity image before  
 applying non-maximum suppression,  $M(i, j)$ . *Right*: The gradient  
 intensity image after applying non-maximum suppression,  $\widehat{M}(i, j)$ .

The choice of the threshold parameters is crucial for the  
 success of the method, so we use the Otsu [43] method to  
 compute these parameters. The method provides the high  
 threshold parameter  $\tau_h$ , and the low threshold parameter  
 $\tau_l$  will be set by us,  $\tau_l = \tau_h/2$ . Pixels that correspond to  
 values higher than the high threshold are considered strong  
 edges and preserved, pixels that fall under the low thresh-  
 old are omitted. Pixels that fall between the two threshold  
 parameters are considered as weak edges and kept only if  
 connected with a strong edge. The last step of the algo-  
 rithm is the connection of already detected edges in the  
 binary image under some restrictions upon the gradient  
 and the gradient angle.

The stages of the edge detection for the three individual  
 defects are shown in Fig. 18, where different colours refer  
 to different stages of the detection. The strong edges de-  
 tected after the non-maximum suppression are coloured in  
 blue. In green and red, are the parts of the edge omitted  
 or added after the hysteresis tracking, respectively. The  
 green part of the edges is the last added part during the  
 connectivity analysis.

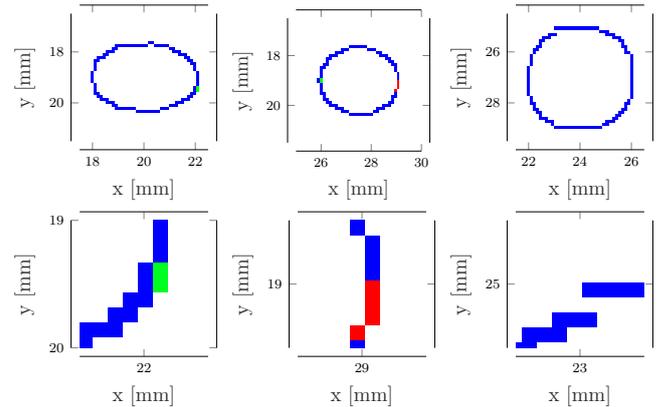


Figure 18: The stages of the edge detection algorithm illustrated  
 with different colours.

The final binary image which gives the edges of the  
 flaws is depicted in Fig. 19 where the true shape of the  
 defects is depicted with green colour lines.

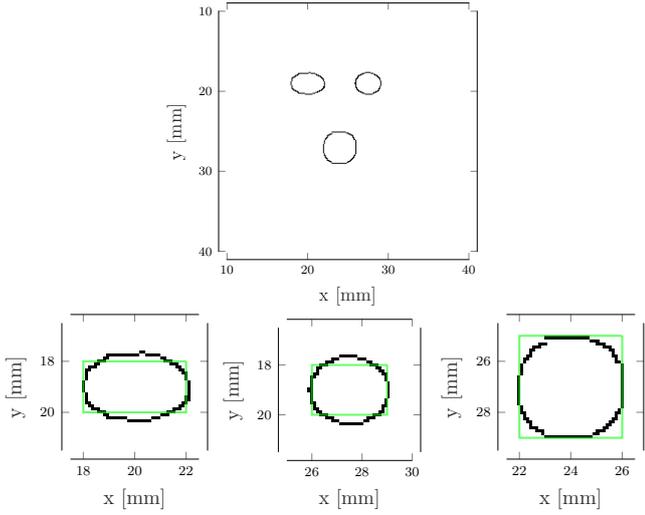


Figure 19: The final binary image.

759 The presented multi-step procedure was fully automated  
 760 exempt the fact that the smoothing parameter  $\sigma$  is needed  
 761 to be given. By giving a different value to this parameter  
 762 the results will change but in our case not dramatically as  
 763 can be seen in Fig. 20, where the smoothed input image is  
 764 given, for  $\sigma = 3$ , and the final reconstructed binary image.

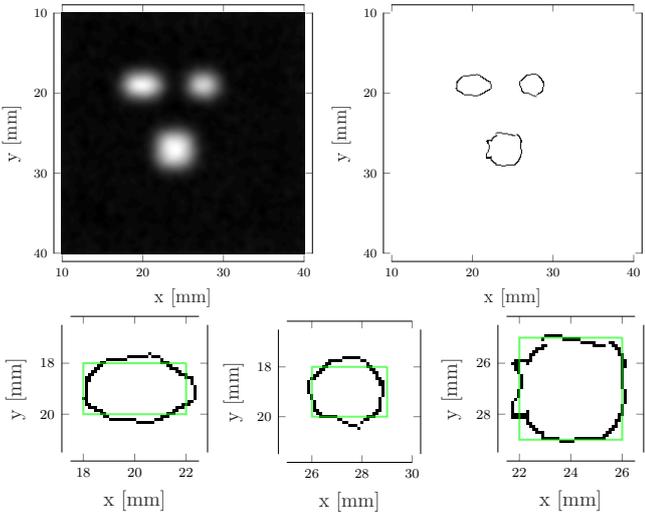


Figure 20: Gaussian smoothing of the input image to the Canny<sup>783</sup>  
 algorithm shown in grey scale. *Left:* The noisy input image  $\mathcal{I}^*$ .<sup>784</sup>  
*Right:* The smoothed image  $\widehat{\mathcal{I}}^*$ .

765

#### 766 4.4. Non uniform heating

767 The previous results have been produced under the as-  
 768 sumption of an ideal homogeneous heating. In reality, a  
 769 number of factors such as the finite camera support, its  
 770 position and orientation, local variations of material prop-  
 771 erties, etc. can introduce gradients into the heat front  
 772 profile. In order to study the impact of this effect on the

773 reconstruction results, the profile of the impinging flux has  
 774 been modified by introducing a gradient along the  $x$ -  
 775 direction as follows

$$J_e(x, y) = - \left[ 1 - \sin(\phi_x) \frac{x - x_1}{x_2} \right] k^{(1)} \frac{\partial \widehat{T}^{(e)}}{\partial z} \quad (55)$$

776 where  $x_1$  and  $x_2$  are the limits of the flash support in  
 777 the  $x$ -direction. This excitation term leads to an uneven  
 778 heating of the piece in the  $x$ -direction, with the left side  
 779 being heated more than the right one.

780 The TSR results for the best contrast image for  $\phi_x = 5$   
 781 and  $\phi_x = 30$  with an SNR equal to 10 dB are shown in  
 782 Fig. 21. The corresponding results of the Canny algorithm  
 for the edge detection are given in Fig. 22.

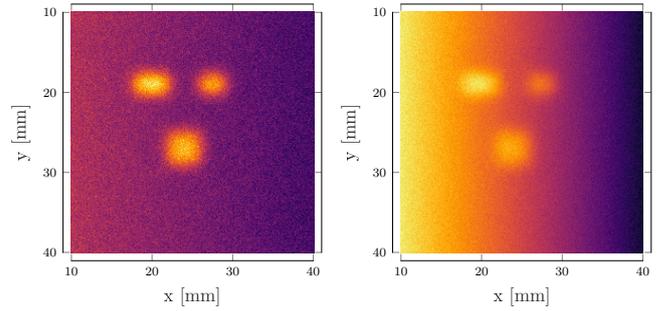


Figure 21: Best TSR reconstructed frame with different heat-front  
 gradient. *Left:*  $\phi_x = 5$ . *Right:*  $\phi_x = 30$

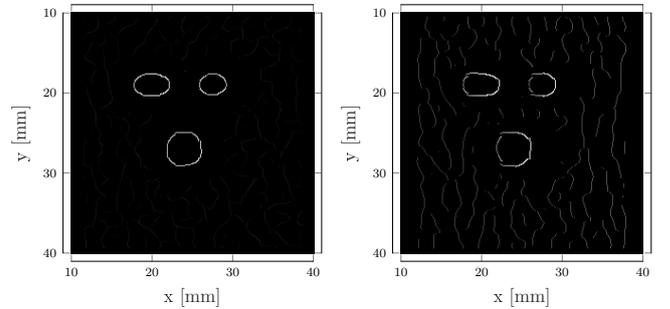


Figure 22: Results for the edge detection obtained using the frames of  
 Fig. 21 after application of non-maximum suppression. *Left:*  $\phi_x = 5$ .  
*Right:*  $\phi_x = 30$

785 Being relatively smooth, the gradient of the thermal  
 786 field does not seem having a severe impact to the recon-  
 787 struction images (cf. Fig. 21). As far as the edge detection  
 788 is concerned, the algorithm seems to be relatively insensi-  
 789 tive, with a weak variation of the edge stroke being visible  
 for the stronger gradient ( $\phi_x = 30$ ).

790 It must be underlined however that the above con-  
 791 clusions apply as far as the field gradient is relatively  
 792 low-frequency, in other words smooth with respect to the  
 793 sought defect. This assumption is met for sources with  
 794 extended illumination support, such as flash lamps con-  
 795 sidered in this work. The algorithm is expected to be less

796 robust when point-wise heating, using Laser sources for<sup>815</sup>  
 797 example, is applied.

#### 798 4.5. Depth effect

799 The reconstruction results are expected to be also de-<sup>819</sup>  
 800 pendent from the depth of the defects, since the primary<sup>820</sup>  
 801 field of the heat source becomes more diffuse. This ef-<sup>821</sup>  
 802 fect is important, when the inspection is carried out from<sup>822</sup>  
 803 the opposite side of the excitation (transmission thermog-<sup>823</sup>  
 804 raphy). In order to test the algorithm performance with<sup>824</sup>  
 805 increasing defect depth, the simulations have been car-<sup>825</sup>  
 806 ried out for three different depths, namely  $z = 0.75\text{mm}$ ,<sup>826</sup>  
 807  $z = 1.5\text{mm}$  and  $z = 2.25\text{mm}$ , and the reconstruction al-  
 808 gorithm has been applied using the corresponding noise-  
 809 corrupted images. The raw and TSR reconstructed results  
 810 for the optimal-contrast frame are shown in Fig. 23. The  
 811 SNR has been set equal to 10 dB.

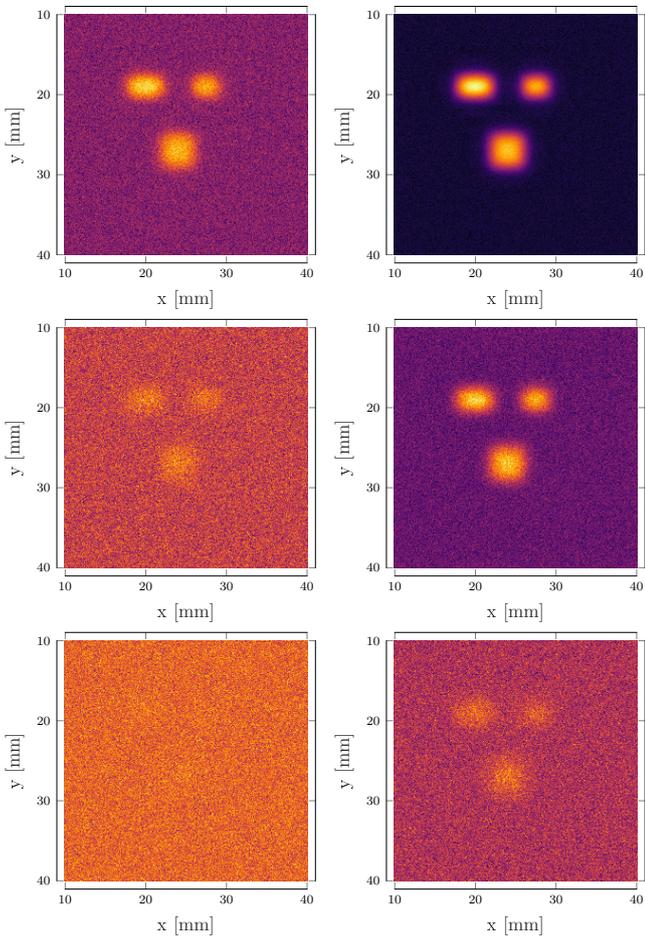


Figure 23: Comparison of a raw image with the reconstructed image  
 corresponding to the same optimal frame number for three different<sup>837</sup>  
 defect depths  $z = 0.75\text{mm}$ ,  $z = 1.5\text{mm}$  and  $z = 2.25\text{mm}$ . *Left*<sup>838</sup>  
*Column:* Raw images. *Right Column:* Reconstructed images.<sup>839</sup>

812 From the illustrated results the beneficial effect of the<sup>841</sup>  
 813 TSR-induced denoising becomes evident and the resulting<sup>842</sup>  
 814 enhancement of the edge detection algorithm is obvious.<sup>843</sup>  
 844

#### 4.6. Complex defect shapes

For a better judgement of the reconstruction capabilities in the case of defects with more complex shapes, the proposed algorithm has been applied in the case of a circular and an irregular shaped defect, whose geometries are given in Fig. 24. In the same figure are illustrated the edges forms obtained after the application of the TSR-Canny approach with non-maximum suppression. Both defects are embedded in the same depth with the one of the nominal case, namely 1.5 mm under the upper surface and the SNR has been set to 10 dB. It appears that the applied approach yields overall satisfactory results for both cases.

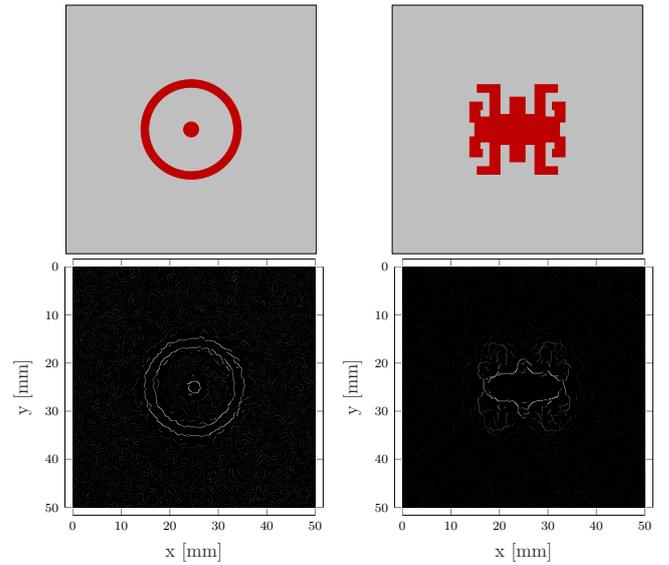


Figure 24: Defect geometries in the  $(x, y)$ -plane and the corresponding edge-detection results after application of the TSR-Canny approach. *Left column:* A disc of radius 1.5 mm included in a ring of internal radius 8.5 mm and external radius 10 mm. *Right column:* A complex shape of dimensions  $20 \times 18$  mm.

## 5. Discussion

The TSR method has been combined with the Canny algorithm for the detection and the dimensioning of delamination defects in thermograms obtained using pulsed thermography. In this combined approach TSR is used for compression and denoising the raw signals, and the thus proceeded images are passed to the Canny algorithm. It is interesting to note the partial spatial denoising of the images (beside the temporal filtering) achieved by the TSR, which contributes to a beneficial improvement to the overall processing thus enhancing the performance of the Canny algorithm.

TSR has been also used as an acceleration technique for the simulation of the heat propagation problem, in the sense that the temperature field needs to be calculated at a limited number of time instances, used for the polynomial fitting when the TSR approximation is computed.

845 Combined with a fast semi-analytical solution based on 910  
846 the TREE approach, TSR provides a powerful tool. 911

847 This work is restricted to the characterisation of delam- 912  
848 ination defects in homogeneous and isotropic media with 913  
849 the thermal excitation being applied upon one of the piece 914  
850 interfaces. Volumetric heat production via ultrasonic of 915  
851 eddy current excitation [47, 48] are of great practical in- 916  
852 terest as well, and work is under way in order to extend 917  
853 the herein developed approach in order to treat such kind 918  
854 of inspection scenarios as well. 919

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