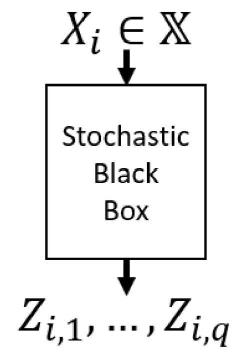
BAYESIAN MULTI-OBJECTIVE OPTIMIZATION WITH NOISY EVALUATIONS

1. CONTEXT

- **Multi-objective optimization** of the parameters of a planning strategy for the multi-year planning of the electricity distribution grid [DUT15]
- A stochastic black box provides **noisy evaluation results** of the objective functions $f_1, \dots, f_q$ defined on a discrete search domain $\mathbb{X} \subset \mathbb{R}^d$
- Previous *n* evaluations at $\mathrm{X} = (\mathrm{X}_1, \dots, \mathrm{X}_\mathrm{n})$ assumed:
  - $Z_{i,1} = f_1(X_i) + \varepsilon_{i,1}, \dots, Z_{i,q} = f_q(X_i) + \varepsilon_{i,q}$
    where $\varepsilon_{i,j}$s are zero-mean random variables



2. OBJECTIVE

Estimate the **Pareto-optimal solutions** (or Pareto set Γ) of the problem:

$$x^\star = \operatorname*{argmin}_{x \in \mathbb{X}} f_1(x), \dots, f_q(x)$$

Defined as:

$$\Gamma = \{x \in \mathbb{X} : \nexists x' \in \mathbb{X} \text{ such that } f(x') \prec f(x)\}$$

where $\prec$ stands for the Pareto domination rule:

$$y = (y_1, \dots, y_q) \prec y' = (y'_1, \dots, y'_q) \quad \Leftrightarrow \quad \begin{cases} \forall i \leq q, y_i \leq y'_i \\ \exists j \leq q, y_j < y'_j \end{cases}$$

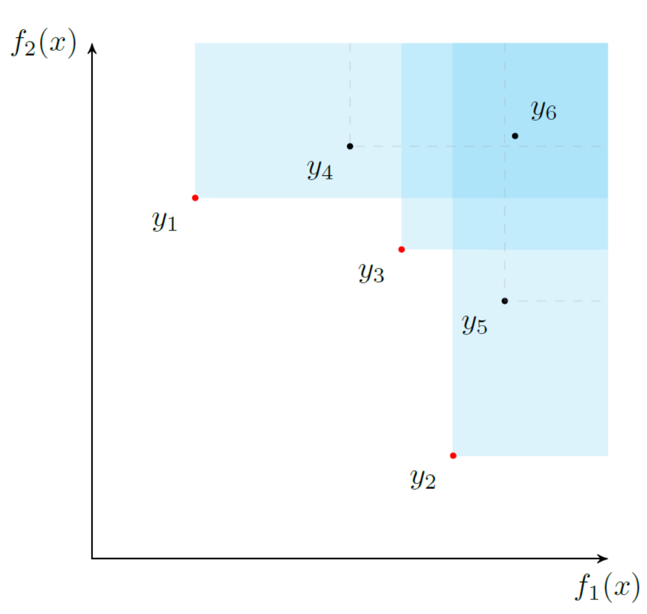
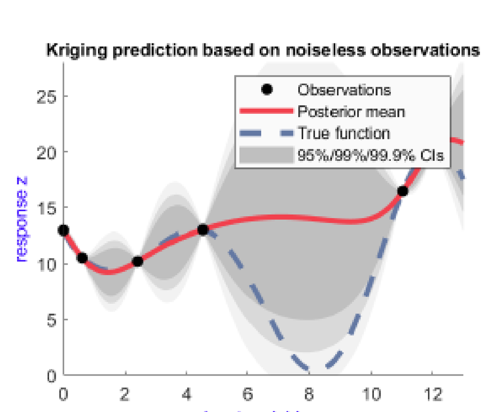


Illustration of the Pareto domination rule
- y_1, y_2, y_3 are non-dominated points
- y_4 is only dominated by y_1
- y_5 is only dominated by y_2
- y_6 is dominated by all other points

3. BAYESIAN OPTIMIZATION

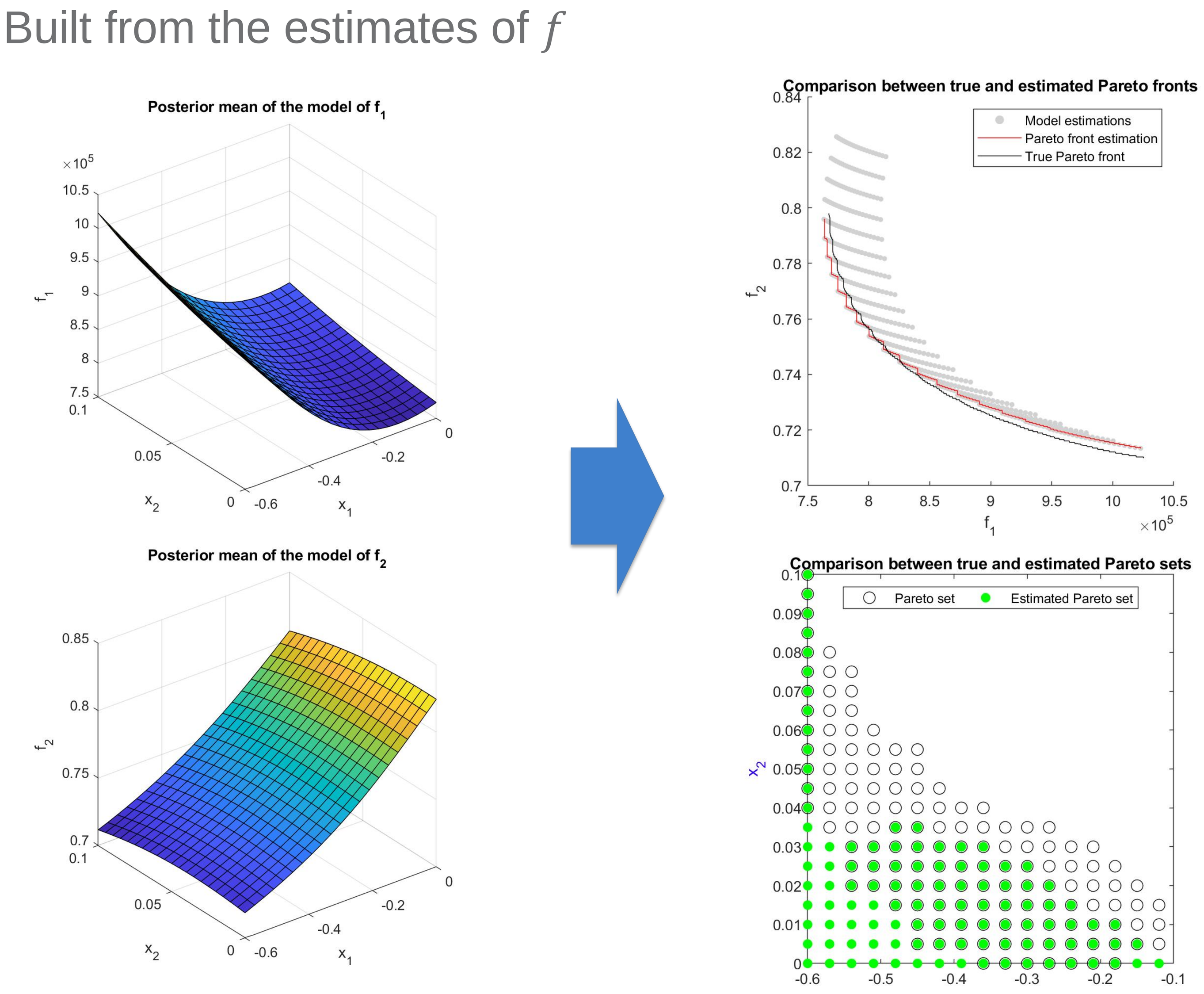
- Define a **probabilistic model** for each f conditional on previous observations
- Use a **sampling criterion** to select new evaluation points



Provides estimate of f and a measure of uncertainty of the estimation

PARETO-OPTIMAL ESTIMATES

Built from the estimates of f



PROPOSED APPROACH

- Replace the multi-objective problem by the minimization of a single **augmented Tchebycheff function** [KNO06]:

$$\tilde{f}(x) = \max_j[\omega_j f_j(x)] + \rho \sum_j \omega_j f_j(x), \qquad \sum_j \omega_j = 1, \rho > 0$$

- At each iteration, generate random weights ω_j and apply this function to the ***n* previous observations**:

$$\tilde{Z}_i = \max_j[\omega_j Z_{i,j}] + \rho \sum_j \omega_j Z_{i,j}, \qquad , i = 1, \dots, n$$

- Assume a homoscedastic Gaussian noise model and fit to $\tilde{Z} = (\tilde{Z}_1, \dots, \tilde{Z}_n)$ a Gaussian Process model $\tilde{\xi}_n$ with parameters estimated by maximum likelihood
- We use **the Knowledge Gradient** (KG) criterion [FRA09] to select new point X_{n+1} based on previous observations. The idea is to identify a point that is expected to reduce the minimum of the posterior mean of $\tilde{\xi}$:

$$X_{n+1} = \operatorname*{argmax}_{x \in \mathbb{X}} KG\,(x)$$

with:

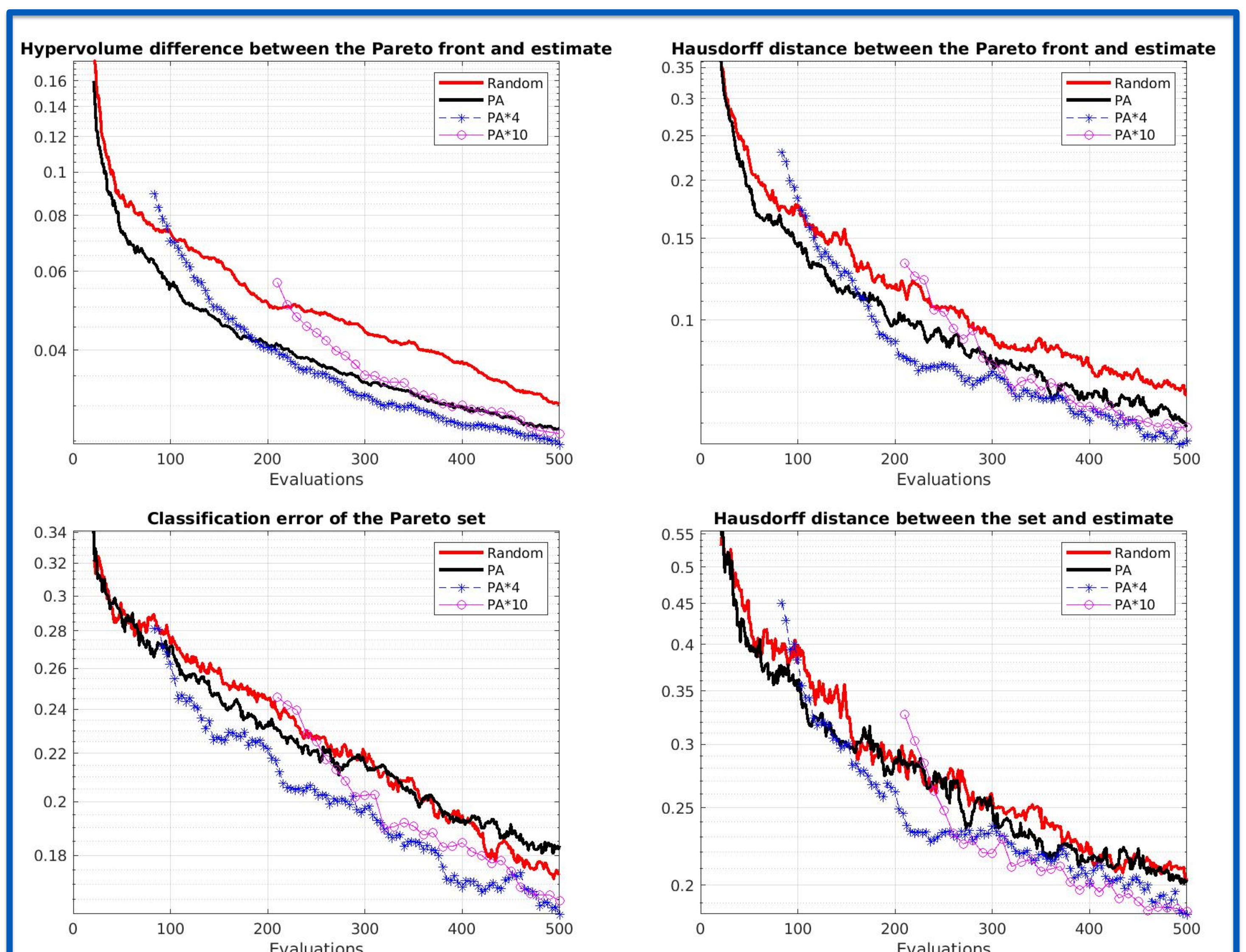
$$KG(x) = \min_{x' \in \mathbb{X}} \mathbb{E}[\tilde{\xi}_n(x')] - \mathbb{E}\left[\min_{x' \in \mathbb{X}} \mathbb{E}[\tilde{\xi}_n(x') | \tilde{Z}_{n,x}]\right]$$

where $\tilde{Z}_{n,x}$ denotes a new observation of $\tilde{\xi}_n$ at x

- Update the model with new observation and iterate until stopping criterion is met

4. NUMERICAL EXPERIMENTS

- Compare the proposed approach (PA) with random selection of points in a bi-dimensional bi-objective problem
- Compare use of batches of 1, 4 or 10 evaluations



Average performance results over 100 simulations for four optimization approaches: random approach (red) and the proposed approach with batches of 1 (black), 4 (blue) or 10 (magenta) evaluations.

5. OPEN QUESTIONS

- **'Ideal' batch of evaluations**?
- **Performance comparison** of the proposed approach to other methods in the literature?

REFERENCES

[DUT15] Dutrieux, H. (2015). "Méthodes pour la planification pluriannuelle des réseaux de distribution. Application à l'analyse technico-économique des solutions d'intégration des énergies renouvelables intermittentes". Doctoral Thesis, Ecole Centrale de Lille.

[FRA09] Frazier, P., Powell, W., and Dayanik, S. (2009). "The knowledge-gradient policy for correlated normal beliefs". INFORMS Journal on Computing, 21(4):599–613.

[KNO06] Knowles, J. (2006). "ParEGO: A hybrid algorithm with on-line landscape approximation for expensive multiobjective optimization problems". IEEE Transactions on Evolutionary Computation, 10(1):50–66.

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