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BAYESIAN MULTI-OBJECTIVE OPTIMIZATION
WITH NOISY EVALUATIONS

1. CONTEXT
• Multi-objective optimization of the parameters of a planning strategy for the multi-year planning of the electricity distribution grid [DUT15]
• A stochastic black box provides noisy evaluation results of the objective functions $f_1, \ldots, f_d$ defined on a discrete search domain $X \subset \mathbb{R}^d$
• Previous $n$ evaluations at $X = (X_1, \ldots, X_n)$ assumed:
  * $Z_{1:i} = f_1(x_i) + \epsilon_{1:i}, \ldots, Z_{i:q} = f_d(x_i) + \epsilon_{i:q}$
  where $\epsilon_{i:s}$ are zero-mean random variables

2. OBJECTIVE
Estimate the Pareto-optimal solutions (or Pareto set $\Gamma$) of the problem:

$$x^* = \arg\max_{x \in \mathbb{R}^d} f(x)$$

where $\prec$ stands for the Pareto domination rule:

$$f(x) \prec f(x')$$

$\Rightarrow \left\{ \begin{array}{l}
\forall i, y_i \leq y'_i \\
\exists j, y_j \leq y'_j \wedge y'_j < y_j
\end{array} \right.$

Illustration of the Pareto domination rule:
- $y_1, y_2, y_3$ are non-dominated points
- $y_1$ is only dominated by $y_2$
- $y_3$ is dominated by all other points

3. BAYESIAN OPTIMIZATION
• Define a probabilistic model for each $f$ conditional on previous observations.
• Use a sampling criterion to select new evaluation points.

Provides estimate of $f$ and a measure of uncertainty of the estimation

PARETO-OPTIMAL ESTIMATES
Built from the estimates of $f$

PROPOSED APPROACH
• Replace the multi-objective problem by the minimization of a single augmented Tchebycheff function [KNO06]:

$$f(x) = \max_{x \in \mathbb{R}^d} \omega f_j(x) + \rho \sum_{j=1}^d \omega_j f_j(x), \quad \sum_{j=1}^d \omega_j = 1, \rho > 0$$

• At each iteration, generate random weights $\omega_j$ and apply this function to the $n$ previous observations:

$$Z_{i+1} = \max_{x \in \mathbb{R}^d} \omega Z_{i:j} + \rho \sum_{j=1}^d \omega Z_{i:j}, \quad i = 1, \ldots, n$$

• Assumes a homoscedastic Gaussian noise model and fit to $Z = (Z_1, \ldots, Z_n)$ a Gaussian Process model $\zeta$ with parameters estimated by maximum likelihood.
• Use the Knowledge Gradient (KG) criterion [FRA09] to select new point $X_{n+1}$ based on previous observations. The idea is to identify a point that is expected to reduce the minimum of the posterior mean of $\zeta$:

$$X_{n+1} = \arg\max_{x \in \mathbb{R}^d} KG(x)$$

with:

$$KG(x) = \min_{x \in \mathbb{R}^d} \mathbb{E}[f(x)] - \mathbb{E}[\min_{x \in \mathbb{R}^d} f(x)]$$

where $Z_{n+1}$ denotes a new observation of $\zeta$ at $x$.

Update the model with new observation and iterate until stopping criterion is met.

4. NUMERICAL EXPERIMENTS
• Compare the proposed approach (PA) with random selection of points in a bi-dimensional bi-objective problem.
• Compare use of batches of 1, 4 or 10 evaluations.

5. OPEN QUESTIONS
• ‘Ideal’ batch of evaluations?
• Performance comparison of the proposed approach to other methods in the literature?

REFERENCES
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