Correlated source localization with orthogonal least squares

Gilles Chardon, José Picheral, François Ollivier

CORRELATED SOURCE LOCALIZATION WITH ORTHOGONAL LEAST SQUARES

Gilles Chardon[1] **José Picheral**[1] **François Ollivier**[2]

[1] Université Paris-Saclay, CNRS, CentraleSupélec,
Laboratoire des signaux et systèmes, 91190, Gif-sur-Yvette, France
[2] Institut Jean le Rond d'Alembert, Sorbonne Université, 75005 Paris, France
gilles.chardon@centralesupelec.fr

ABSTRACT

In addition to estimating the positions of acoustical sources, estimating their mutual correlations can yield important informations about the sources. A simple example is the pairing of a source and its images in a reverberant environment, as they are perfectly correlated. Jointly estimating the positions and correlations of sources is a computational challenge, both in memory and time complexity since the entire covariance matrix of the sources has to be recovered (and not only its diagonal as for standard powers estimation). Correlated sources are also known to prevent the application and subspace-based methods such as MUSIC. We propose to estimate the covariance matrix of the sources in a greedy way, using the Orthogonal Least Squares algorithm. This algorithm allows efficient identification of the sources, with reasonable computational requirements. The performances of the method are demonstrated with experimental measurements, using correlated sources (multiple sources emitting the same signal, or a unique source with a reflection).

1. INTRODUCTION

Source imaging is an important problem of acoustical signal processing, with applications in industrial acoustics, source separation, medical imaging, etc. Several methods have been proposed to solve this problem, such as the well-known delay-and-sum (DAS) beamformer, Bayesian estimation methods, or sparsity based techniques. The focus of this communication is the joint estimation of the positions (or directions) of acoustical sources with their covariance matrix. In other words, our goal is to localize, and identify sources that are correlated. Applications include pairing of sources with their reflections in reverberant environments, and imaging of distributed sources.

Correlated sources is a challenge for subspace-based methods such as MUSIC or ESPRIT. Indeed, in this case where the covariance matrix of the sources is rank-deficient, and the signal subspace has a lower dimension that the number of the sources.

Here, the correlation of the sources is not considered a nuisance, but a characteristic of the sources to be estimated. Estimation of the covariance of the sources is a numerical challenge. Indeed, if the space is discretized using L points, the dimension of the covariance matrix of the sources is $L \times L$. It is here necessary to design a low-complexity algorithm, if possible, linear in the size of the grid. Indeed, a quadratic algorithm would already have a complexity $O(L^4)$, implying extremely long computations even for moderate size problems.

Estimation methods for the covariance matrix of sources have been proposed, such as a variant of the DAS beamformer, limited by its resolution, or convex optimization based methods. These proposed methods have complexity $O(L^6)$, or $O(L^3 L_h^3)$, where L_h is an upper bound on the number of independent sources.

The method we propose, named CMF-OLS for Covariance Matrix Fitting by Orthogonal Least Squares [1] [1] , is based on the greedy sparse recovery algorithm OLS [2, 3], similar to the more popular Orthogonal Matching Pursuit (OMP) algorithm [4, 5]. Using the particular structure of the covariance matrix to be estimated, the complexity of the algorithm is linear in the size of the discretization, allowing its application to large-scale problems. Also, its only parameter is the number of sources, which can be estimated by inspecting the convergence of the algorithm. Individual sources can be extracted from the covariance matrix after its estimation.

2. PROBLEM AND STATE OF THE ART

We assume that the microphone array providing the $N \times 1$ measurements $\mathbf{x}(t)$ contains N elements. The position of the sources are assumed to be described by a parameter Θ, discretized on a grid $\{\Theta_\ell\}_{\ell=1,\dots,L}$. For narrowband sources, the measurements can be decomposed as

$$\mathbf{x}(t_i) = \sum_{\ell=1}^{L} \mathbf{a}(\Theta_\ell) s_\ell(t_i) + \mathbf{n}(t_i), \quad (1)$$

where $\mathbf{a}_l = \mathbf{a}(\Theta_\ell)$ is the $N \times 1$ is the steering vector modeling the propogation between the position Θ_l and the array, $s_l(t_i)$ is the amplitude of the source l at instant t_i modeled as a random variable, and $\mathbf{n}(t_i)$ models the noise, assumed spatially white and gaussian with variance σ^2.

[1] Figures used in this paper are reprinted with permission from [1].

With the $N \times L$ matrix $\mathbf{A} = [\mathbf{a}_1, \ldots, \mathbf{a}_L]$ formed by the L steering vectors corresponding to the L candidate sources of the predefined grid, the signal model can be rewritten as

$$\mathbf{x}(t_i) = \mathbf{A}\mathbf{s}(t_i) + \mathbf{n}(t_i), \tag{2}$$

where $\mathbf{s}(t_i)$ contained the amplitudes of the sources $s_l(t_i)$, with only a few nonzero coefficients.

Assuming that the noise and the sources are independent, the covariance matrix $\mathbf{\Gamma}$ of the measurements is decomposed as

$$\mathbf{\Gamma} = E\{\mathbf{x}(t_i)\mathbf{x}(t_i)^H\} = \mathbf{A}\mathbf{C}\mathbf{A}^H + \sigma^2\mathbf{I}, \tag{3}$$

where $\mathbf{C}$ is the $L \times L$ covariance matrix of the sources.

In practice, the covariance matrix $\mathbf{\Gamma}$ is estimated using the sample covariance matrix of a set of I measurements:

$$\mathbf{G} = \frac{1}{I}\sum_{i=1}^{I} \mathbf{x}(t_i)\mathbf{x}(t_i)^H \approx \mathbf{\Gamma}. \tag{4}$$

Our goal is to estimate the covariance matrix of the sources $\mathbf{C}$ from the knowledge of the estimated covariance matrix of the measurements $\mathbf{G}$.

2.1 State of the art

Several estimation methods have been proposed for this problem, mostly based on convex optimization. A representative example is the Sparse Spectrum Fitting method [6, 7], where the covariance matrix is estimated as the solution of the following optimization problem

$$\widehat{\mathbf{C}}_{SpSF} = \arg\min_{\mathbf{C}} \|\mathbf{G} - \mathbf{A}\mathbf{C}\mathbf{A}^H\|_F^2 + \lambda\|\mathbf{C}\|_1 \quad \text{s.t.} \quad \mathbf{C} \geq 0. \tag{5}$$

where the ℓ_1 norm promotes sparsity of the estimation, which is also constrained to be symmetric positive semi-definite. Variants include a modification of the problem aiming at the joint estimation of the noise level [8], or an additional term promoting low-rank matrices [9]. Computation complexity of these methods is in the order of L^6, making them unusable for even moderately sized problems.

The MACS method was also proposed [10]. Replacing a convex problem by a non-convex problem in a lower dimensional space, the complexity is reduced to $O(L^3 L_h^3)$, where L_h is a bound on the number of sources, still too high for large scale problems.

3. GREEDY ALGORITHM FOR CORRELATED SOURCE LOCALIZATION

The method we propose, based on the Orthogonal Least-Squares algorithm [2], aims at identifying the sources in an iterative way, i.e. incrementing a set S of columns such that

$$\mathbf{G} \approx \mathbf{A}_S\mathbf{C}\mathbf{A}_S^H, \tag{6}$$

The algorithm will operate as follows :

- Initialization: $k = 1$, residual $\mathbf{R}_0 = \mathbf{G}$, and set of indices of identified sources $S_0 = \emptyset$.
- Identification of an additional source of index $l^\star$, $S_k = S_{k-1} \cup \{l^\star\}$
- Update of the residual $\mathbf{R}_k$, by removing the contribution of the identified sources to the covariance matrix
- $k \leftarrow k + 1$, iterate until a stopping criterion is met (number of sources, norm of the residual, etc.).

Sources are identified by projecting the residual $\mathbf{R}_k$ on the space of the covariance matrices that can be generated by the set of sources S_k augmented by a candidate source l. The source $l^\star$ maximizing the Frobenius norm of the projection is added to the set of sources.

The residual is updated by taking the projection of the covariance matrix on the space of covariance matrix generated by this new set of sources S_{k+1}. Elementary linear algebra calculations show that the orthogonal projection of a matrix $\mathbf{M}$ on the space of covariance matrices generated by a set of sources S is found as:

$$\Pi_S(\mathbf{M}) = \mathbf{A}_S^\perp \mathbf{A}_S^{\perp H} \mathbf{M} \mathbf{A}_S^\perp \mathbf{A}_S^{\perp H} \tag{7}$$

where $A_S^\perp$ is the matrix of an orthogonal basis of the space spanned by the vectors $\mathbf{a}(\Theta_l)$ for $l \in S$.

The complexity of the agorithm is $O(LN^2K)$, where L is the size of the grid, N the number of microphones, and K the number of steps of the algorithm. Its memory footprint is dominated by the size of the dictionary LN. We emphasize here the fact that the computation complexity is linear with respect to the size of the grid.

Extensive discussion of the algorithm and additional mathematical and implementation details are found in [1].

4. EXPERIMENTAL RESULTS

We present here some experimental results, highlighting the performances of the method. A planar array of 128 MEMS microphones is used, placed in a disk of radius 1.5m with four sources (see figure 1), at a distance of 4.3m from the array in an anechoic chamber (see figure 2). Results for two experiments are given here, with the following setup:

- two pairs of correlated sources (visible on figure 3),
- two sources, with a planar reflector.

Measurements are filtered around the frequency $f = 2.2$kHz, and the region of interest is a 2m×2m rectangle, discretized using $L = 400 \times 200 = 8\text{e}4$ points. Additional experiments and extensive discussion are given in [1].

4.1 Two pairs of coherent sources

Results of the DAS beamformer and the two groups of sources identified by CMF-OLS are pictured on figure 4. At this frequency, the DAS beamformer is unable to separate the two closely spaced sources. In constrast, the four

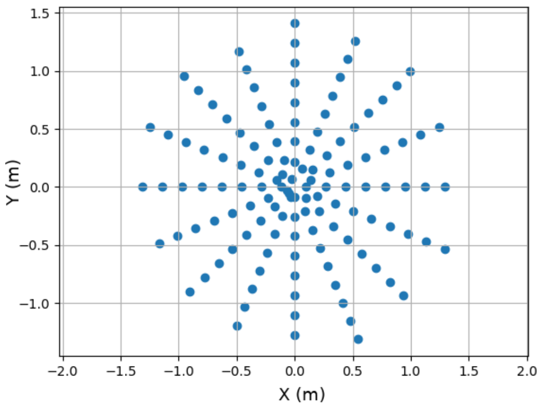

Figure 1. Microphone array layout.

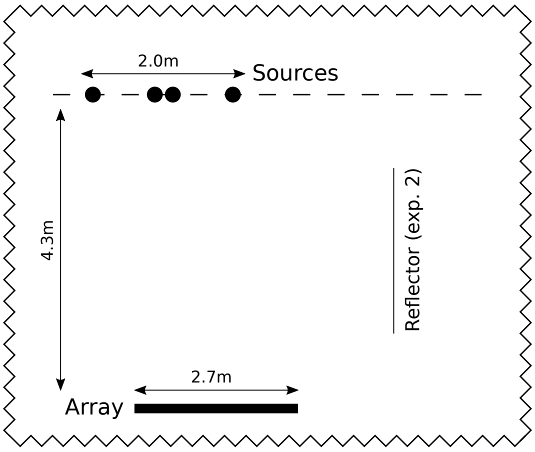

Figure 2. Setup of the experiment.

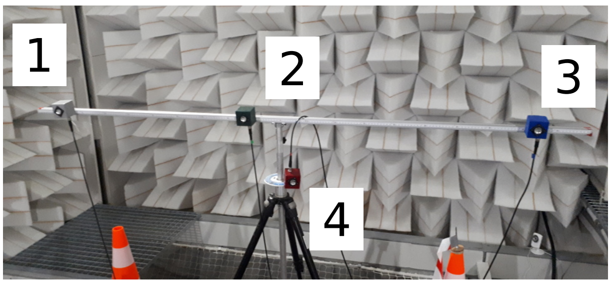

Figure 3. Sources layout.

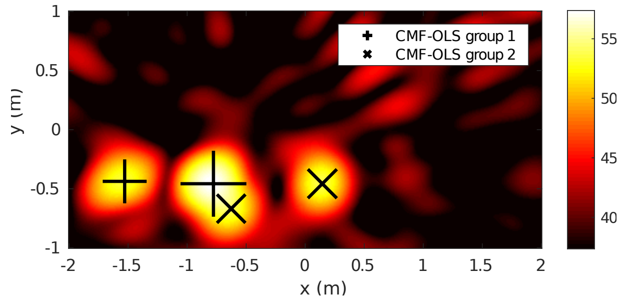

Figure 4. Results of the CMF-OLS algorithm and output of the DAS beamformer. Two pairs of correlated sources

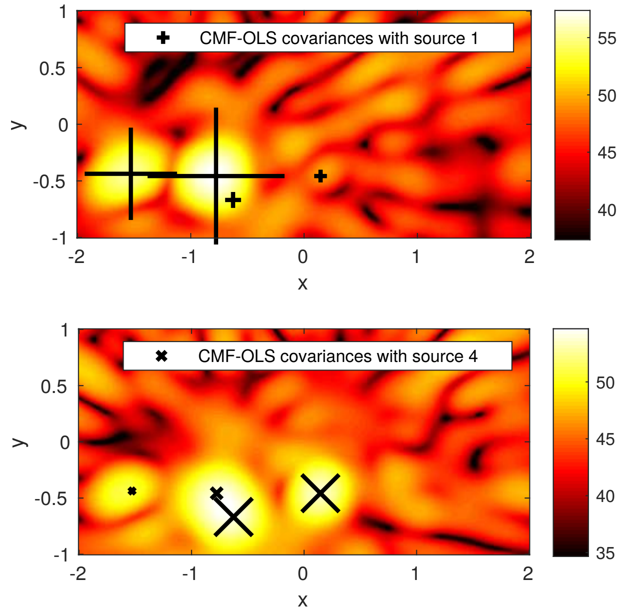

Figure 5. Results of the CMF-OLS algorithm and output of the DAS beamformer. The two identified blocks of sources are given, with the covariances estimated by the DAS beamformer.

sources are correctly localized by CMF-OLS, and correctly grouped in pairs of correlated sources.

Figure 5 shows the two groups of sources separately. the CMF-OLS results are superimposed over the covariances estimated by the beamformer.

4.2 Two sources and a reflector

In a second experiment, a planar reflector is introduced in the anechoic chamber. It is expected here that two image sources appear, symmetric to the actual sources with respect to the reflector, completely correlated with their respective original sources.

The results of fig. 6 fulfill the expectations: two pairs of correlated sources are identified, with positions symmetric with respect to the reflector. Here, $f = 2.93$kHz , the scan area has dimensions 7m$\times 1.5m$, including the virtual source space, discretized over $L = 1.05e5$ points.

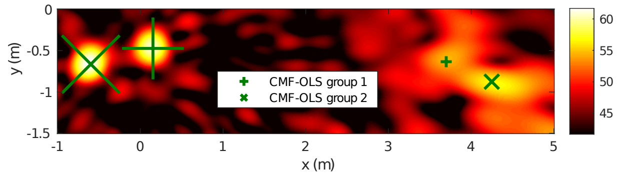

Figure 6. Results of the CMF-OLS algorithm and output of the DAS beamformer. Two sources with reflections.

5. CONCLUSION

A numerically efficient algorithm for the joint localization of acoustical sources and estimation of their correlations is introduced. The low computational complexity of the method allows application to large-scale problems.

6. REFERENCES

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