Disturbance Frequency Estimation for an LTV System

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Abstract: We consider the frequency estimation problem for a sinusoidal disturbance acting on a linear time-varying system, where only the input and output signals are available. We propose a novel parametrization that translates this problem into a linear regression model with unknown parameters. The frequency estimation is performed using the Dynamic Regressor Extension and Mixing procedure and an algebraic finite-time estimator. The resulting scheme provides finite-time frequency estimation under the interval excitation condition. The role of the tuning coefficients in satisfying the convergence condition is also discussed. Simulations illustrate the applicability of the proposed solution.

Keywords: Adaptive system and control, Frequency estimation, Time-varying systems, Excitation

1. INTRODUCTION

The disturbance frequency estimation problem has a broad scope of engineering application. Notably, besides the disturbance compensation and indirect adaptive regulation, this problem also arises in fault detection (Alcorta-Garcia et al., 2011) and marine applications (Belleter et al., 2015). The most common problem statement considers a linear time-invariant (LTI) plant perturbed by a periodic, mono- or multi-sinusoidal disturbance (Aranovskiy et al., 2009; Wang et al., 2017). Solutions to this problem are typically based on the plant's frequency response, and a linear time-invariant transformation is applied to measurements generating auxiliary instrumentals signals oscillating with the same frequencies as the disturbance. Such an instrumental signal is often referred to as an *image* of the disturbance (Landau et al., 2013). When the image of the disturbance is generated, the frequency estimation problem can be solved with a method suitable for *measured* signals. A variety of such methods exist, e.g., notch filters (Mojiri and Bakhshai, 2004), PLL-based methods (Fedele and Ferrise, 2014; Pin et al., 2014), second-order generalize integrators (Fedele et al., 2009), and others, including methods for signals with time-varying parameters (Vedyakov et al., 2017); see (Aranovskiy et al., 2016b) for a comparison of selected methods.

The LTI problem statement described above is motivated by linearizing a plant around an equilibrium, e.g., stabilization. On the other hand, if the plant follows a trajectory, its linearization may result in a linear time-varying (LTV) system. Given that the system is subject to a sinusoidal perturbation, the disturbance frequency identification problem is formulated for an LTV model. Unfortunately, as the frequency response analysis does not apply to time-varying systems, the discussed approach is unsuitable. Besides some particular methods, e.g., for periodic systems (Louarroudi et al., 2014), the problem of frequency estimation for LTV systems with unmeasured states remains widely open.

This paper proposes a solution suitable for a mono-sinusoidal disturbance acting on a system with the relative degree (with respect to the disturbance) equal to one. The proposed solution is based on a novel parameterization applied to the considered LTV system. This parameterization yields a linear regression equation (LRE), where the unknown frequency appears as an element of the vector of unknown parameters. Thus, the frequency identification problem is transformed into the parameter estimation problem of an LRE.

The LRE estimation is one of the fundamental problems in adaptive control, and numerous solutions are available. Two standard solutions are the Least Squares with Forgetting and the Gradient algorithms (Sastry and Bodson, 2011; Narendra and Annaswamy, 2012; Ioannou and Sun, 2012). Both methods, widely used in adaptive control and estimation, imply the Persistency of Excitation assumption. Periodic disturbances acting on LTI systems typically satisfy these requirements; however, it becomes less straightforward for an LTV system. As the system's trajectories depend not only on the input signals but also on the time-varying system's parameters, the excitation propagation analysis may be complicated. Motivated by this reason, we aim at the frequency estimation under the relaxed requirement of Interval Excitation.

The parameter estimation problem under relaxed excitation requirements attracts researchers' attention, and multiple solutions are available, such as composite (Pan and

⋆ This work was partially supported by the Ministry of Science and Higher Education of Russian Federation, passport of goszadanie no. 2019-0898, and by Rennes Metropole, France (project AIS-19C0326).

Yu, 2018) and concurrent (Chowdhary and Johnson, 2010) learning, excitation prolongation (Korotina et al., 2022), fixed- and finite-time estimators (Wang et al., 2020), to name a few. It is worth noting that the parametrization proposed in this paper generates a vector of unknown parameters having more elements than required for the frequency estimation. This observation motivates the use of the Dynamic Regressor Extension and Mixing (DREM) procedure (Aranovskiy et al., 2016a) that allows element-wise estimation of unknown parameters. In order to ensure estimation under the relaxed excitation condition, a finite-time algebraic estimator is then applied (Ortega et al., 2020a).

Novelty and contribution. The paper proposes a novel parametrization allowing for disturbance frequency estimation for an LTV system. The proposed parametrization allows for a DREM-based estimator providing the finite-time frequency estimation under the interval excitation. Moreover, we also discuss and illustrate the role of tuning coefficients in the imposed convergence condition.

This article is organized as follows. In Section 2, we formulate our problem statement. Section 3 presents the novel parametrization yielding a linear regression model, and Section 4 discusses parameter estimation under the interval excitation condition. Simulation results are presented in Section 5. The paper is wrapped up with the conclusion discussing possible extensions of this work.

Notation:

- $\mathbb{R}$ is the set of reals, $\mathbb{R}_+$ is the set of nonnegative reals, and $\mathbb{N}$ is the set of positive integers;
- I_n is the $n \times n$ identity matrix, for all $n \in \mathbb{N}$ (when clear form the context, the subscript n can be omitted);
- $p = \frac{d}{dt}$ is the differential operator;
- a linear operator $\mathcal{F}$ applied to a signal $x : \mathbb{R}_+ \to \mathbb{R}$, we denote as $\mathcal{F}[x(t)]$; if $x : \mathbb{R}_+ \to \mathbb{R}^n$, then $\mathcal{F}[x(t)]$ denotes the element-wise application of the operator;
- the argument of time may be omitted when clear from the context.

2. PROBLEM STATEMENT

Consider a linear time-varying system given by

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t) + G\sigma(t), \\ y(t) &= C^\top x(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $y(t) \in \mathbb{R}$ is the measured output signal, $u(t) \in \mathbb{R}^{n_u}$ is the measured input, $A(t)$ and $B(t)$ are the state and the input time-varying matrices of the corresponding dimensions, respectively, $C \in \mathbb{R}^{n_x}$ and $G \in \mathbb{R}^{n_x}$ are constant matrices, and

$$\sigma(t) = A_\sigma \sin(\omega t + \phi_\sigma) \tag{2}$$

is the unknown and unmeasured disturbance with nonzero frequency $\omega > 0$. The matrices $A(t)$, $B(t)$, C, and G are known, the state x is not measured. The relative degree with respect to the disturbance σ equals to one; this condition is formulated in the following Assumption.

Assumption 1. The matrices C and G are such that $C^\top G \neq 0$.

The goal is to estimate the unknown frequency ω of the disturbance σ given the measurements of u and y, i.e. to construct an estimate $\hat{\omega}(t)$ such that

$$\lim_{t\to\infty} |\hat{\omega}(t) - \omega| = 0.$$

As discussed in Introduction, the key obstacle to frequency estimation in (1) is that the frequency-domain tools are not pertinent for time-varying systems, and the output signal is not necessarily periodic with the same frequency as the disturbance. Thus, most available approaches do not apply, and a more sophisticated parametrization is required to translate the frequency estimation goal into the parameter estimation problem.

Moreover, this excitation does not necessarily propagate in time-varying systems despite sinusoidal signals being exciting. Thus, it is desirable to estimate the unknown frequency in the absence of the persistent excitation, e.g., under the weaker assumption of the interval excitation.

3. PARAMETERISATION

This section develops a parametrization that translates the frequency estimation problem to the parameter estimation for a linear regression model. Towards this end, we first introduce several auxiliary signals and variables, and then we use these definitions to present the parametrization, which is the main result of this section.

3.1 Auxiliary Signals

Define the matrix $F(t) \in \mathbb{R}^{n_x \times n_x}$ and the vector $V(t) \in \mathbb{R}^{n_x}$ as

$$F(t) := A(t)\left(I - \frac{1}{C^\top G} G C^\top\right), \tag{3}$$

$$V(t) := B(t)u(t) + \frac{1}{C^\top G} A(t) G y(t). \tag{4}$$

Define also the auxiliary signals $\nu(t) \in \mathbb{R}^{n_x}$ and $\chi(t) \in \mathbb{R}$ as

$$\nu(t) := \left(-C^\top P(t)\right)^\top, \tag{5}$$

$$\chi(t) := y(t) - C^\top \xi(t), \tag{6}$$

where $P(t) \in \mathbb{R}^{n_x \times n_x}$ and $\xi(t) \in \mathbb{R}^{n_x}$ are the solutions of

$$\dot{P}(t) = F(t)P(t), \quad P(0) = I_{n_x}, \tag{7}$$

$$\dot{\xi}(t) = F(t)\xi(t) + V(t), \quad \xi(0) = 0, \tag{8}$$

respectively.

Define the stable second-order filter

$$\mathcal{F} := \frac{\lambda^2}{(p+\lambda)^2}, \tag{9}$$

where $\lambda > 0$ is the tuning coefficient. Applying $\mathcal{F}$ and $p^2\mathcal{F}$ to the previously introduced signals ν and χ, define

$$q(t) := p^2\mathcal{F}\left[\chi(t)\right], \tag{10}$$

$$m(t) := \begin{bmatrix} \mathcal{F}\left[\chi(t)\right] \\ p^2\mathcal{F}\left[\nu(t)\right] \\ \mathcal{F}\left[\nu(t)\right] \end{bmatrix}, \tag{11}$$

where $q(t) \in \mathbb{R}$ and $m(t) \in \mathbb{R}^{2n_x+1}$.

3.2 Proposed Parametrization

Using the notation introduced in Section 3.1, we are now in the position to formulate the following Theorem.

Theorem 1. Along the trajectories of (1), the signals $q(t)$ and $m(t)$ defined in (10) and (11), respectively, satisfy the linear regression equation

$$q(t) = m^\top(t)\theta + \epsilon_\lambda(t), \tag{12}$$

where $\theta \in \mathbb{R}^{2n_x+1}$ is the vector of unknown constant parameters,

$$\theta := \begin{bmatrix} -\omega^2 \\ e_0^\top \\ \omega^2 e_0^\top \end{bmatrix}$$

with $e_0 \in \mathbb{R}^{n_x}$ defined as

$$e_0 := -x(0) - G\frac{A_\sigma}{\omega}\cos(\phi_\sigma), \tag{13}$$

and ϵ_λ is a generic exponentially decaying term whose rate of decay depends on the tuning coefficient λ.

Proof. For the sinusoidal signal (2) it holds

$$\ddot{\sigma}(t) = -\omega^2\sigma(t).$$

Define the new variable

$$z(t) := x(t) + G\frac{1}{\omega^2}\dot{\sigma}(t). \tag{14}$$

From (1) and (14) it follows

$$y(t) = C^\top\left(z(t) - G\frac{1}{\omega^2}\dot{\sigma}(t)\right), \tag{15}$$

and

$$-\frac{1}{\omega^2}\dot{\sigma}(t) = \frac{1}{C^\top G}\left(y(t) - C^\top z(t)\right), \tag{16}$$

where $C^\top G \neq 0$ due to Assumption 1.

Then the time derivative of z is given by

$$\begin{aligned}\dot{z}(t) &= A(t)x(t) + B(t)u(t) + G\sigma(t) + G\frac{1}{\omega^2}\ddot{\sigma}(t) \\ &= A(t)x(t) + B(t)u(t).\end{aligned}$$

Adding and subtracting the term $AG\frac{1}{\omega^2}\dot{\sigma}$ and recalling (16), the derivative of $z(t)$ can be written as

$$\begin{aligned}\dot{z}(t) &= A(t)\left(x(t) + G\frac{1}{\omega^2}\dot{\sigma}(t)\right) + B(t)u(t) - A(t)G\frac{1}{\omega^2}\dot{\sigma}(t) \\ &= A(t)z(t) + B(t)u(t) + A(t)G\frac{1}{C^\top G}\left(y(t) - C^\top z(t)\right) \\ &= F(t)z(t) + V(t),\end{aligned}$$

where $F(t)$ and $V(t)$ are defined in (3) and (4), respectively. Note that the initial condition

$$z(0) = x(0) + G\frac{A_\sigma}{\omega}\cos(\phi_\sigma)$$

is not known.

Define $e := \xi - z$, where ξ is defined in (8). Then

$$e(0) = -z(0) = e_0,$$

where e_0 is defined in (13). The time derivative of $e(t)$ is thus

$$\dot{e}(t) = F(t)e(t). \tag{17}$$

Since $P(t)$ defined in (7) is the fundamental matrix of the LTV (17), it holds

$$e(t) = P(t)e_0,$$

and

$$z(t) = \xi(t) - P(t)e_0.$$

The latter allows rewriting (15) as

$$\chi(t) = \nu^\top(t)e_0 + d(t), \tag{18}$$

where ν and χ are defined in (5) and (6), respectively, and

$$d(t) := -\frac{C^\top G}{\omega^2}\dot{\sigma}(t).$$

Note that since σ is a sinusoidal signal, it holds

$$\ddot{d}(t) = -\omega^2 d(t) = -\omega^2\left(\chi(t) - \nu^\top(t)e_0\right).$$

Apply $p^2\mathcal{F}$ to the both sides of (18), where $\mathcal{F}$ is defined in (9), and note that

$$p^2\mathcal{F}[d(t)] = \mathcal{F}\left[\ddot{d}(t)\right] + \epsilon_\lambda(t),$$

where $\epsilon_\lambda(t)$ is a generic exponentially decaying term whose rate of decay depends on the tuning coefficient λ. Then

$$p^2\mathcal{F}[\chi(t)] = \mathcal{F}\left[\ddot{d}(t)\right] + \left(p^2\mathcal{F}[\nu(t)]\right)^\top e_0 + \epsilon_\lambda(t),$$

and thus

$$\begin{aligned}p^2\mathcal{F}[\chi(t)] = -\omega^2\mathcal{F}[\chi(t)] &+ \left(p^2\mathcal{F}[\nu(t)]\right)^\top e_0 \\ &+ \omega^2\left(\mathcal{F}[\nu(t)]\right)^\top e_0 + \epsilon_\lambda(t),\end{aligned}$$

which can also be written as (12). □

Remark 1. Theorem 1 translates the frequency estimation problem to the parameter estimation problem for the linear regression model (12). The number of unknown parameters in (12) is $2n_x + 1$. However, for the frequency estimation, we are only interested in the first element of the vector θ, namely $\theta_1 = -\omega^2$. That motivates the use of the DREM procedure in Section 4 because it allows estimating a single parameter and not the whole vector θ.

3.3 Excitation conditions

The system (1) is excited by the sinusoidal signal σ. However, it is not a trivial task to analyze how this excitation propagates through the proposed parametrization. Indeed, the regressor m in (12) depends on the behavior of the time-varying matrices A and B, and the persistency of excitation of m can hardly be ensured in advance. On the other hand, it is reasonable to expect that the regressor m is exciting at least during transients in the system (1). For the parameter estimation task, we further assume that the regressor m possesses the Interval Excitation property, which is formulated in the following definition.

Definition 1. (Interval Excitation). A bounded signal $x : \mathbb{R}_+ \to \mathbb{R}^n$ is exciting on the interval $[t_1,\, t_1 + t_c]$ if for some $t_1 \geq 0$ and $t_c > 0$ there exists $\alpha > 0$, such that

$$\int_{t_1}^{t_1+t_c} x(s)x^\top(s)ds \geq \alpha I_n.$$

This property is further denoted as x being (t_1, t_c, α)-IE or $x \in$ IE.

Remark 2. The interval excitation can also be referred to as *sufficient excitation* (Kamalapurkar et al., 2017). Also, the interval excitation property with $t_1 = 0$ is known as *initial excitation* (Pan and Yu, 2018).

Assumption 2. The vector m defined in (11) is (t_1, t_c, α_m)-IE for $t_1 = 0$ and some $t_c > 0$, $\alpha_m > 0$.

4. PARAMETER ESTIMATION

This section presents an approach to estimate the frequency ω of the disturbance σ given the linear regression model (12). Since we are interested in estimating only the first element of the vector θ, namely $\theta_1 = \omega^2$, we apply the DREM procedure, which translates the vector problem into a set of scalar estimation problems for each element of the vector of unknown parameters independently and preserves the excitation properties. Further, as we assume only the Interval Excitation of the regressor m in (12), we combine the DREM procedure with a finite-time algebraic observer. The resulting scheme allows for frequency estimation under a proper choice of tuning parameters.

4.1 DREM procedure

The DREM procedure consists of two steps, where the first step is the *dynamics extension*, and the second step is *mixing*. At the first step, the linear regression model (12) is extended to obtain a square regression matrix. There exist various possible dynamics extension method as discussed by Ortega et al. (2020b). Among these methods, Kreisselmeyer's extension scheme is of particular interest as it guarantees the interval excitation preservation (Korotina et al., 2020).

For the LRE (12), Kreisselmeyer's scheme is given by

$$\dot{\Phi}_{\mathcal{H}}(t) = -a\Phi_{\mathcal{H}}(t) + \beta m(t)m^\top(t), \quad \Phi_{\mathcal{H}}(0) \geq 0, \tag{19}$$

$$\dot{Y}_{\mathcal{H}}(t) = -aY_{\mathcal{H}}(t) + \beta m(t)q(t), \tag{20}$$

where $a > 0$ and $\beta > 0$ are the tuning coefficients. Here $\Phi_{\mathcal{H}}(t)$ is a symmetric square $(2n_x+1)\times(2n_x+1)$ matrix, and $Y_{\mathcal{H}}(t)$ is a vector with $2n_x+1$ elements; the initial value $Y_{\mathcal{H}}(0) \geq 0$ can be arbitrary. Then $\Phi_{\mathcal{H}}$ and $Y_{\mathcal{H}}$ form the extended linear regression equation

$$Y_{\mathcal{H}} = \Phi_{\mathcal{H}}\theta, \tag{21}$$

with the same vector θ as (12).

The *mixing* step translates the extended equation (21) into a set of scalar equations. Multiplying (21) on the left by the adjugate matrix of $\Phi_{\mathcal{H}}$, we obtain

$$\mathcal{Y}_i(t) = \Delta(t)\theta_i, \ i \in \{1, 2, ..., 2n_x+1\},$$

where

$$\mathcal{Y} := \mathrm{adj}\{\Phi_{\mathcal{H}}\}Y_{\mathcal{H}},$$

and

$$\Delta = \det\{\Phi_{\mathcal{H}}\}.$$

In what follows, we are interested only in the equation for $i = 1$, namely

$$\mathcal{Y}_1(t) = \Delta(t)\theta_1, \tag{22}$$

as it allows estimating θ_1 and thus ω.

It is worth also noting that due to the excitation preservation properties of Kreisselmeyer's scheme, the new regressor Δ is also exiting on the same interval as the original regressor m, i.e. Δ is $(0, t_c, \alpha_\Delta)$-IE for some $\alpha_\Delta > 0$.

4.2 Finite-time estimation

To estimate θ_1, we use the Finite-Time Convergence (FTC) estimator (Ortega et al., 2020a). Recall first the standard gradient estimator given by

$$\dot{\hat{\theta}}_1(t) = \gamma\Delta(t)\left[\mathcal{Y}_1(t) - \Delta(t)\hat{\theta}_1(t)\right] \tag{23}$$

for some initial value $\hat{\theta}_1(0)$, where $\gamma > 0$ is the tuning coefficient, and $\hat{\theta}_1(t)$ is the estimate of θ_1.

For the gradient estimator (23), the condition $\Delta \in \mathrm{IE}$ is not sufficient for the convergence of $\hat{\theta}_1(t)$ to the real value θ_1. To this end, we apply the FTC estimator

$$\hat{\theta}_1^{\mathrm{FTC}}(t) := \frac{1}{1-\kappa^c(t)}\left[\hat{\theta}_1(t) - \kappa^c(t)\hat{\theta}_1(0)\right], \tag{24}$$

where $\hat{\theta}_1$ is given by (23) and $\kappa^c(t)$ is the clipping function

$$\kappa^c(t) = \begin{cases} \mu, & \text{if} \quad \kappa(t) \geq \mu, \\ \kappa(t), & \text{if} \quad \kappa(t) < \mu, \end{cases}$$

with $\mu \in (0,1)$ and $\kappa(t)$ given by

$$\dot{\kappa}(t) = -\gamma\Delta^2(t)\kappa(t), \ \kappa(0) = 1.$$

The convergence of the FTC estimator (23)-(24) under the interval excitation is formulated in the following proposition.

Proposition 2. Consider the linear regression equation (22) and the estimation algorithm (23)-(24). Let Δ be $(0, t_c, \alpha_\Delta)$-IE in the sense of Definition 1 for some $t_c > 0$ and $\alpha_\Delta > 0$. If parameters $\gamma > 0$ and $\mu \in (0,1)$ are such that the inequality

$$\gamma\,\alpha_\Delta \geq -\ln(\mu), \tag{25}$$

is satisfied, then the parameter estimation $\hat{\theta}_1^{\mathrm{FTC}}(t)$ converges to the real value θ_1 in finite time:

$$\hat{\theta}_1^{\mathrm{FTC}}(t) = \theta_1, \ \forall t \geq t_c. \tag{26}$$

The proof of Proposition 2 is given in (Ortega et al., 2020a, Proposition 6).

Remark 3. It is worth noting that the excitation level α_Δ of the new regressor Δ depends both on the excitation level α_m of the original regressor m in (12), see Assumption 2, but also on the parameters a and β of Kreisselmeyer's scheme (19), (20). For more details on this relation see (Aranovskiy et al., 2022).

5. SIMULATION

Consider the system (1) with $n_x = 2$,

$$A(t) = \begin{bmatrix} -1 & \cos(0.1t) \\ -\cos(0.1t) & -0.5 \end{bmatrix}, \quad B = \begin{bmatrix} \cos(2t) \\ \frac{1}{1+t} \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad G = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, \tag{27}$$

Choose the input signal $u = \sin(t)$ and the disturbance

$$\sigma(t) = 0.3\sin(\omega_1 t), \ \omega_1 = 3. \tag{28}$$

The output signal $y_1(t)$ is depicted in Fig. 1.

For the parameterization filter (9), the tuning parameter $\lambda = 30$ is chosen. For the DREM procedure (19)–(20), we choose $\Phi_{\mathcal{H}}(0) = 0$, $Y_{\mathcal{H}}(0) = 0$, $a = 0.5$, and $\beta = 1$. The new regressor Δ is depicted in Fig. 2; the figure illustrates that the regressor Δ satisfies the Interval Excitation requirement.

For the estimation algorithms (23) and (23)–(24), we choose the tuning coefficients $\gamma = 1$ and $\mu = 0.95$. As $\theta_1 = \omega^2$, the estimate of ω is constructed as

$$\hat{\omega}(t) = \sqrt{|\hat{\theta}_1(t)|},$$

$$\hat{\omega}^{\mathrm{FTC}}(t) = \sqrt{|\hat{\theta}_1^{\mathrm{FTC}}(t)|}.$$

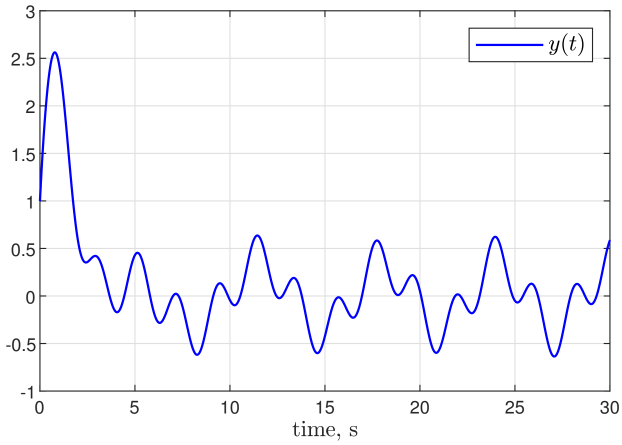

Fig. 1. The output signal $y(t)$ in the system (1), (27), (28).

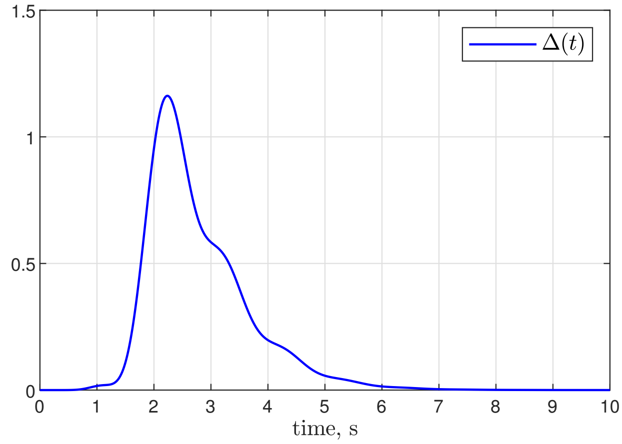

Fig. 2. The new regressor Δ.

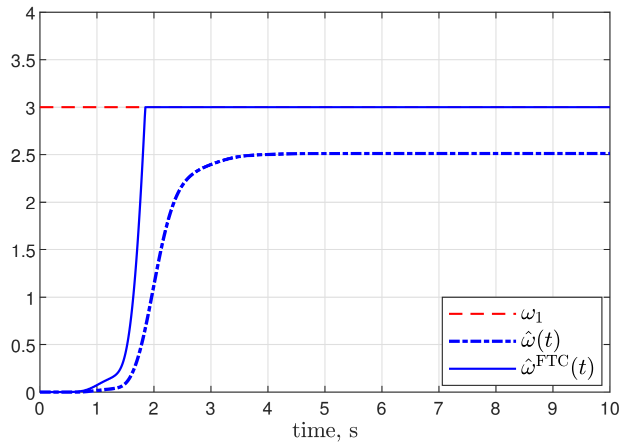

Fig. 3. Estimates of ω_1 using (23) and (23)–(24).

The estimation results are shown in Fig. 3. Due to the weak excitation of Δ, the gradient algorithm (23) does not converge, and the estimate $\hat{\omega}$ does not reach the real value $\omega_1 = 3$. However, for the FTC estimator (23)–(24), the convergence is achieved.

Consider now the impact of various tuning coefficients on the estimation convergence. Towards this end, replace the disturbance $\sigma(t)$ given by (28) with

$$\sigma(t) = 0.01 \sin(\omega_2 t + \pi/8), \quad \omega_2 = 7. \tag{29}$$

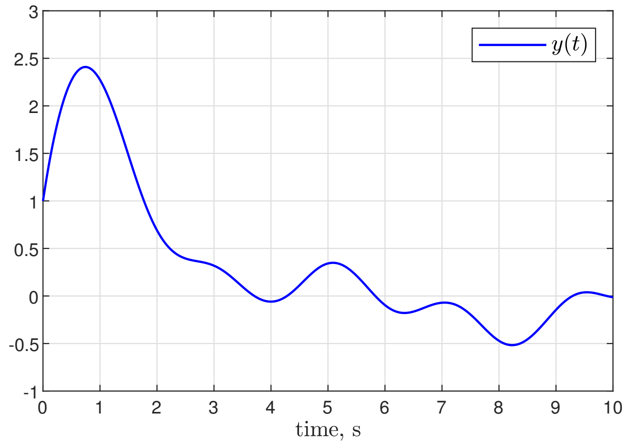

Fig. 4. The output signal $y(t)$ in the system (1), (27), (29).

The output signal $y(t)$ corresponding to the new disturbance is depicted in Fig. 4.

The convergence of the FTC estimator (23)–(24) depends on the condition (25). It can be shown that the previously used tuning coefficients do not satisfy the condition (25) for the regressor $\Delta(t)$ resulting from the disturbance $\sigma(t)$ given by (29).

The retuning of the coefficients can be performed increasing the gains γ and μ, or by adjusting the value of α_Δ via the parameter β of (19)–(20). The increase of μ for values close to one may induce undesirable numerical implementation effects as μ affects the expression $1 - \kappa^{\mathrm{c}}(t)$ in the denominator of (26). Therefore, we consider tuning of the coefficient γ and adjustment of the value α_Δ via the coefficient β.

In (Aranovskiy et al., 2022), the excitation preservation properties of (19)–(20) are studied, and it is shown that the value α_Δ grows as $\beta^{2(2n_x+1)}$, i.e., small changes in β allow for significant increase of α_Δ.

Specifically, in this simulation example, the convergence condition (25) is satisfied for the disturbance (29) setting $\beta = 2$. In contrast, the required value of the coefficient γ is 10^6, cf. with $\beta = 1$ and $\gamma = 1$ for the disturbance (28). To summarize, β is the preferable tunning coefficient impacting the convergence condition (25). The resulting convergence of the FTC estimator for $\beta = 2$ and $\gamma = 1$ is shown in Fig. 5.

6. CONCLUSION

A method is proposed to estimate the unknown frequency of a sinusoidal disturbance acting on an LTV system. As the frequency-response analysis does not apply, a novel parameterization procedure is developed to translate the frequency estimation into the constant parameters vector estimation for a linear regression model.

The desired parameter is estimated using the DREM procedure with an algebraic finite-time estimator. The resulting scheme ensures the frequency estimation under the interval excitation condition subject to a proper choice of tuning coefficients. The necessary and sufficient condition for the estimator convergence is given, and the impact of the tuning coefficients is discussed.

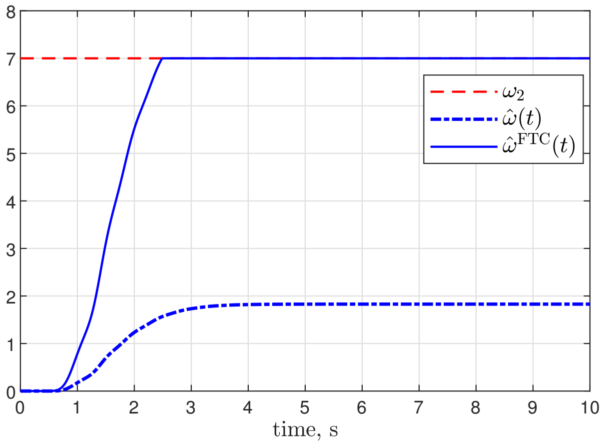

Fig. 5. Estimates ω_2 estimations using (23) and (23)–(24).

The proposed solution considers the frequency estimation problem for a single disturbance. The further extension of the proposed method is to consider the case of a multi-sinusoidal disturbance. Our preliminary studies indicate that the proposed parametrization can be generalized to the case when the number of sinusoidal components in the signal $\sigma(t)$ is known.

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