A Resource Allocation Algorithm for Formation Control of Connected Vehicles

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***Abstract*— A formation control problem is considered for a network of connected vehicles communicating over a 5G C-V2X communication system with limited network resources. To solve this problem, we propose a new combined control - resource allocation scheme to make the vehicles achieve the targeted formation. We concentrate on the formulation of the formation control problem under the constraints imposed by the communication system. We state the resource allocation problem and present an optimization algorithm to select the transmitting agents. The proposed algorithm allows to assign the network resources in a centralized way to ensure the convergence towards the desired formation while preserving the network connectivity.**

***Index Terms*— Consensus algorithm, Formation control, Resource allocation, 5G communication, Optimization.**

I. Introduction

THE consensus problem has been widely studied in recent years due to the variety of its applications in many fields such as mobile robots [1], connected and automated vehicles [2], unmanned aerial vehicles [3], social networks [4] ... The consensus algorithm aims to make inter-communicating agents achieve a common value such as the center of a formation in the problem of formation control [1] or a common velocity in platoons of connected and automated vehicles [2].

In the control theory literature, the stability properties of consensus algorithms are studied under strong assumptions on the communication system. Most of the time, the network topology is assumed to be undirected and/or fixed in time [5], [6]. However, in real-world applications, the network topology changes over time and depends on the states of the agents. For example, when dealing with connected vehicles, the network topology depends on their positions since a vehicle can communicate only within a specific range. The Hegselmann-Krause (HK) model proposes an enhanced version of the consensus algorithm presented in [6] to take into account the state-dependence of the network topology [7], [8]. The HK model is generally presented in discrete-time where, at each step, each agent updates its state based on the states of its close neighbors, i.e., agents that belong to some vicinity of the considered agent. In both discrete and continuous time, the HK model provides interesting properties on the network topology that allow to achieve consensus [9], [10]. However, it is assumed that all agents can always communicate with their neighbors. In real communication systems, communication links can be interrupted at some moments due to packet losses. Moreover, the network resources are limited. Therefore, agents can transmit information only if there are available resources allocated to them during a specific period.

The fifth generation of cellular networks (5G) standards provide a framework for vehicular communications known as cellular vehicle-to-everything (C-V2X) [11], [12]. 5G wireless technology is meant to deliver higher peak data rates, lower latency, improved reliability, massive network capacity, increased availability, and a more uniform user experience. Higher performance and improved efficiency empower new user experiences and pave the way to more vertical applications. C-V2X uses cellular connectivity to send and receive signals from a vehicle to other vehicles, pedestrians, or fixed stations such as traffic lights in its surroundings. It offers several tools that enable, for example, road traffic optimization, safely level improvement and reduction of energy consumption [13], [14].

Compared to previous cellular standards like 4G-LTE, the 5G technology allows more flexibility in the design of application-oriented communication systems by providing optimized technologies to implement C-V2X communications. For example, for road-traffic optimization applications, it is crucial that vehicles are able to communicate with each-others in order to agree on a common target such as the vehicles speed or the inter-vehicular distance. In this context, the consensus algorithm represents an efficient tool to make the inter-connected vehicles agree on a common value. The concept of designing jointly the communication and control is explored in [15] and [16] for vehicles platooning. In this work, we present a method to tune the communication system configuration to optimize a criterion related to the control of an arbitrary formation of vehicles, which includes platoons. We propose a resource allocation algorithm that guarantees convergence to the desired formation by preventing the split of the formation that may occur because of the loss of connectivity.

The paper is organized as follows. First, we propose a formulation similar to the continuous-time HK model for the formation control problem in a 5G C-V2X communication

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context. We focus on the communication system's model and the constraint of limited network resources. In the second part of the letter, a resource allocation algorithm is presented along with a characterization of the initial conditions ensuring convergence. In section IV, a simulation example is given to assert the relevance of the proposal. Finally, we conclude by outlining some perspectives for future works.

II. Problem formulation

Consider a network of connected and automated vehicles communicating over a 5G C-V2X communication network. Vehicles are connected to a base station (gNB) and, as we describe in Section II-B, can communicate directly with each other through the C-V2X sidelink. Each vehicle is identified by its position in a 2-dimensional space $p_i \in \mathbb{R}^2$ for $i \in \mathcal{I}_N = \{1, ..., N\}$. The network structure depends on the vehicles' positions. Information is exchanged between vehicles through radio signals. A vehicle can successfully receive the transmitted signal only if it is within the radio coverage of the transmitting vehicles. The radio coverage, also called transmission range (distance), depends on the power of the transmitted signal and the radio conditions. This distance is assumed identical for all vehicles and is denoted as ρ. The communication links are determined by the connectivity function w, defined by:

$$w(p_i, p_j) = \begin{cases} 1; \text{ if } \|p_i - p_j\| \leq \rho \\ 0; \text{ otherwise} \end{cases} \tag{1}$$

Function $w(\cdot, \cdot)$ is an indicator of the presence of a communication link between two agents. This link is symmetric, i.e., if an agent i can receive from agent j, then agent j can also receive from agent i. In this case, i and j are called neighbors.

A. The Formation Control Problem for connected vehicles

There are many applications in which the vehicles are required to follow a predefined trajectory while maintaining a specific formation. For example, in the case of platooning, the vehicles need to travel at a specific speed while forming a line with equal inter-vehicle distances. We suppose that the agents goal is to track a specific trajectory generated by the dynamics $\dot{p}_i(t) = u_{ref}(t)$ while maintaining a formation defined by the offsets with respect to the center of formation. The reference trajectory is assumed to be given by an external controller and known by all agents. The center of formation is a virtual point that is used as a reference to express the shape of the formation in terms of fixed vectors called offsets $d_i \in \mathbb{R}^2$. It can be associated to the average position $\bar{p} = \frac{1}{N}\sum_{i=1}^{N} p_i$ or to the position of a specific agent, for example agent 1, which implies $d_1 = 0$. The goal of the formation control algorithm is to guarantee

$$\lim_{t\to\infty} \|(p_i(t) - p_j(t)) - (d_i - d_j)\| = 0; \ \forall i, j \in \mathcal{I}_N \tag{2}$$

This condition states that the positions of every couple of agents $(i, j) \in \mathcal{I}_N^2$ tends to the desired separation $(d_i - d_j)$ in the targeted formation [17]. Note that the relevant information in formation control problems is the separation vectors rather than the offset vectors.

In order to solve the formation control problem, we propose the following algorithm

$$\begin{cases} \dot{p}_i &= u_i \\ u_i &= u_{ref} + \sum_{j=1}^{N} w(p_i, p_j) \left[(p_j - d_j) - (p_i - d_i)\right] \end{cases} \tag{3}$$

where $u_i \in \mathbb{R}^2$ represents the control law of the i-th agent. u_i is composed of two control parts: the first part is the reference control input that indicates the nominal behavior of the vehicle, and the second part is a consensus-like control input that aims to maintain the shape of the formation. The position of a vehicle is updated according to the positions of its neighbors.

We introduce the relative positions $x_i = p_i - d_i$ for $i \in \mathcal{I}_N$ as the state of i-th agent. The state variables $x_i(t)$ tend to the center of formation when time tends to infinity. Using the new notations, the proposed algorithm is written as follows.

For any $i \in \mathcal{I}_N$,

$$\dot{x}_i = u_{ref} + \sum_{j=1}^{N} w(p_i, p_j)(x_j - x_i) \tag{4}$$

Equation (4), shows that the dynamics of x_i depends on both the actual positions and the relative positions. This is an important difference between our model and the standard formulation of the HK model [7]. Note that we can substitute $w(p_i, p_j)$ by $w(x_i + d_i, x_j + d_j)$ in order to have a more homogeneous expression.

Then, the convergence (consensus) condition (2) can be written as

$$\lim_{t\to\infty} \|x_i - x_j\| = 0; \ \forall i, j \in \mathcal{I}_N$$

B. The formation control problem under a 5G communication framework

In addition to the traditional downlink (DL) and uplink (UL) transmissions between the base station (gNB) and the user equipments (UEs), the 3GPP 4G and 5G cellular systems also enable device-to-device (D2D) communications through the so-called sidelink (SL). We focus here on transmissions between vehicular agents (V2V) over the 5G New Radio (NR) Sidelink that supports two modes of resources allocations, namely Mode 1 and Mode 2 [18].

We focus here on Mode 1 where the gNB assigns and manages the SL radio resources for V2V communications using the DL/UL interfaces. Mode 1 uses dynamic grant (DG) scheduling or configured grant (CG) scheduling that extends the semi-persistent scheduling (SPS) of LTE V2X Mode 4. For clarity purposes, we will use the terminology of SPS in its general meaning to refer to the scheduling mechanism where resource allocation is performed periodically and the same resources are maintained by users for a specific duration.

While the 3GPP standard defines the principle of Mode 1 scheduling and the signaling required to establish and maintain communication links, the method used to select which communication resources to allocate to each UE remains implementation-specific. One of the main purposes of the scheduling is to allocate the less noisy frequencies to UEs

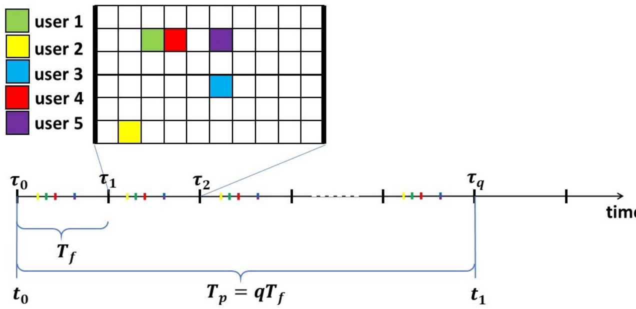

Fig. 1. Structure of a frame and a super-frame: the transmission bandwidth is divided in a grid of time and frequency resource blocks (RBs).

in order to optimize the channel usage while ensuring to all UEs a fair access to resources. However, the scheduling can be configured in order to optimize various criteria related to the application performance. In our work, we propose a scheduling strategy that aims to accelerate the convergence of the vehicles system to the desired formation.

The time-continuity assumption:

We assume here that all agents operate in Mode 1 with SPS scheduling. It is further assumed that all agents transmit with the same periodicity over synchronous periods of time called hereafter frames. In this letter, we use the word "frame" to refer to a logical frame repeating periodically over time, which is different from the physical frame of duration 10ms specified in the 5G standards [12].

The information transmitted by agents is encoded and mapped in an RB as presented in Fig. 1. Thus, the flow of transmitted information is not continuous over time, but transmissions occur at specific instants. Indeed, when a base station allocates an RB to a user i, then user i transmits periodically its state x_i at the corresponding sub-frames. As illustrated in Fig. 1, the duration between two consecutive transmissions corresponds to the frame duration T_f and the number of transmissions corresponds to the number q of frames per super-frame. A super-frame is composed of q consecutive frames, and its duration is given by $T_p = qT_f$.

To simplify the system model, assume that the number of frames per super-frame is sufficiently large $q \gg 1$. In doing so, we can assume that the information transmission is continuous over time. Note that, despite the continuous-time transmission, the communication link between a transmitter and a receiver can be interrupted during a super-frame if the receiver leaves the transmission range.

Let $\{t_k\}_{k\in\mathbb{N}}$ be the sequence of starting times of the super-frames such that t_0 corresponds to the initial time, and $t_{k+1} - t_k = T_p$ for any $k \in \mathbb{N}$.

For any $i \in \mathcal{I}_N$ and $t \in [t_k, t_{k+1})$

$$\dot{x}_i = u_{ref} + \sum_{j=1}^{N} h_{k,j} w(p_i, p_j)(x_j - x_i) \tag{5}$$

where $h_{k,j} \in \{0, 1\}$ for any $j \in \mathcal{I}_N$ and $k \in \mathbb{N}$ such that $h_{k,j} = 1$ if agent j is transmitting at step k and $h_{k,j} = 0$ otherwise. Index j represents the agent number and index k corresponds to the index of the super-frame which is incremented at the beginning of every super-frame.

Note that the value of parameters $h_{k,j}$ in (5) is fixed during the interval $[t_k, t_{k+1})$. Thus, the new system model (5) includes hybrid-dynamics, i.e., continuous-time dynamics and discrete-time dynamics [19]. The control law of $h_{k,j}$ is given by the resource allocation algorithm proposed in Section III. This decision is made at the level of the communication system (the base station gNB) and transmitted to the users, at the beginning of every super-frame (t_k), to parameterize their controllers u_i. This aspect justifies the terminology of joint design of control and communication systems.

The limited resources constraint:

In the case of the sidelink or D2D communication, if two close enough UEs use the same resource, interference may occur leading to packet losses. On the other hand, two UEs can use the same resource if they are far away from each other. This possibility is used in cellular networks to re-use radio resources. Assume that all UEs have the same coverage, i.e., the maximum distance at which a receiver can successfully receive the transmitted information. When a UE needs to transmit a packet, it competes with all other UEs within its coverage. If there are fewer resources per frame than the number of UEs in the coverage region, one has to select which UEs can transmit.

The proposed algorithm aims to assign available resources to users that are allowed to transmit. Let M be the number of available resources per frame. Locally, the number of transmitting users cannot exceed M. However, when two transmitting users are far away from each other, their signals can not interfere due to power loss. Then, they are allowed to use the same resource if it is guaranteed that there is no receiver in the common range of these two transmitting users. In this context, we define a resource reuse distance R as shown in Fig. 2. For example, user u_j belongs simultaneously to the set of neighbors of u_i and u_k. If the same resource is associated to users u_i and u_k then, an interference will occur at the level of u_j. Therefore, in order to ensure an interference-free communication, the resources reuse distance should respect the following condition:

$$R > 2\rho$$

For each agent $i \in \mathcal{I}_N$, define the set of potentially interfering users $\varphi_i = \{j \in \mathcal{I}_n; \|p_j - p_i\| \leq R\}$.

The limited resources constraint can be formulated by the following inequality

$$\sum_{j\in\varphi_i} h_{k,j} \leq M; \ \forall t \in [t_k, t_{k+1}), \forall i \in \mathcal{I}_N \tag{6}$$

This constraint ensures that in a circle of radius R around a given agent, the number of transmitting users cannot exceed the number of resources per frame. Thus, the communication system is guaranteed to be interference-free. Note that φ_i is a time-varying set that depends on the users positions p_i. Since the positions vary over time, then the content of sets φ_i varies over time too.

The transmission indicators change over time and are set by the base station gNB at the beginning of each super-frame. In

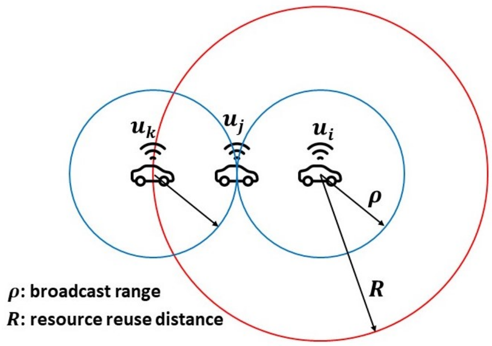

Fig. 2. Illustration of the resource reuse distance. Users belonging to a circle of radius R cannot be associated to the same resource.

the next section, we present the algorithm that allows selecting the transmitting agents during each super-frame.

III. The Resource Allocation Algorithm

The proposed algorithm aims to allocate available network resources per super-frame to users in order to accelerate the speed of convergence to consensus. The algorithm is performed by the base station at the beginning of each super-frame. It makes it possible to select the transmitting users while ensuring that the limited resources constraint is respected. Since the number of transmitting users is lower than the number of available resources, the 5G system will be able to attribute the resources to users and ensure an interference-free communication during the convergence to the desired formation.

To characterize the distance to the desired formation, i.e., achieving consensus, we use the cost function

$$V(x) = \frac{1}{2}\sum_{i,j=1}^{N} \|x_i - x_j\|^2 \tag{7}$$

This function is positive definite on the consensus set, i.e.,

$$V(x) = 0 \iff x_1 = ... = x_N$$

In the next section, we present an iterative algorithm that minimizes the cost function V at the end of each super-frame under the agents' dynamics and the communication system constraints. Let us first recall some notations about graph theory.

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be an undirected graph of order N, where $\mathcal{V} = \{v_i; i \in \mathcal{I}_N\}$ and $\mathcal{E} = \{e_{i,j} = (v_i, v_j); i, j \in \mathcal{I}_N\}$ are respectively the set of vertices and the set of edges. There is an edge between v_i and v_j if $e_{i,j} \in \mathcal{E}$. A path between v_i and v_j is the sequence of edges $\{e_{i_1,i_2}, e_{i_2,i_3}, ..., e_{i_{s-1},i_s}\}$ where $v_{i_1} = v_i$, $v_{i_s} = v_j$ and $e_{i_l,i_{l+1}} \in \mathcal{E}$ for any $l = 1, .., s-1$. We say that a graph is connected if for any $i, j \in \mathcal{I}_N$, there is a path between v_i and v_j. The notion of graph connectivity will be used in the formulation of the proposed algorithm.

Let $p = (p_1, ..., p_N)^T \in \mathbb{R}^{2N}$ be the vector containing the vehicles positions. We denote by $\mathcal{G}(p) = (\mathcal{V}(p), \mathcal{E}(p))$ the graph composed of vertices in positions p_i. An edge $e_{i,j} \in \mathcal{E}(p)$ if $\|p_i - p_j\| \leq \rho$, or equivalently $w(p_i, p_j) = 1$, where $w(\cdot, \cdot)$ is the connectivity function defined in (1).

Note that, in formation control problems, it is implicitly assumed that the graph formed by vehicles is connected. Indeed, when the graph is unconnected, i.e., the graph is split into clusters that do not have shared communication links, these clusters cannot be attracted to each other without the need of external dynamics.

Therefore, we impose the following condition to ensure that convergence to the desired formation is possible

$$\mathcal{G}(p(t)) \text{ is connected} \quad \forall t \in [t_k, t_{k+1}). \tag{8}$$

The controller that we propose for system (5) has two levels. During each super-frame, the control input is defined by (3) while the transmission indicators $h_{k,l}$ during this super-frame are defined via the following algorithm.

A. Transmission protocol design

For the k-th super-frame, at time t_k, the base station uses its knowledge of the system dynamics to predict the system's behavior over the interval $[t_k, t_{k+1})$ and sends the transmission instructions to users after selecting the optimal transmission configuration $h_k^* = (h_{k,1}^*, ..., h_{k,N}^*)^T \in \{0,1\}^N$ that minimizes the value of the cost function at the end-time of the super-frame $V(t_{k+1})$. The optimization problem is stated as follows

$$h_k^* = \underset{h_k \in \{0,1\}^N}{\operatorname{argmin}} \; V(x(t_{k+1})) \tag{9}$$
$$\text{subject to } (5), (6), (8)$$

The first two constraints correspond to the dynamics of the agents given in (5) and the limited resources constraint (6). The third constraint ensures that the connectivity is preserved during the whole super-frame. Similar to the standard Krause model, it can be shown for model (5) that if the connectivity is lost at some moment t, it is lost for all future time. For this reason, we need to check the graph connectivity only at the end-time of the super-frame t_{k+1}. This property allows us to replace the third constraint by "$\mathcal{G}(p(t_{k+1}))$ is connected" which is equivalent to checking that the second smallest eigenvalue of the Laplacian matrix associated to graph $\mathcal{G}(p(t_{k+1}))$ is strictly positive. This modification allows reducing the algorithm complexity.

The formulated optimization problem fits in the class of Boolean programming problems. Various methods, such as the Branch-and-Bound method [20], are proposed in the literature to handle such problems and can be used to solve (9). Rather than finding the best method for solving the optimization problem (9), we focus in this paper on the convergence properties of the closed loop system (5)-(9).

B. Feasibility of the transmission protocol

Even though the proposed optimization problem guarantees the existence of at least one feasible solution $h_k = (0, ..., 0)^T$, this solution is not satisfactory since it means that there is no information transmission between users. In this case, only the reference dynamics u_{ref} would contribute to the vehicles control and thus the desired formation will not be achieved.

To guarantee the existence of a non-zero feasible solution, we impose assumptions on the graph connectivity at initial time instant $t_0 = 0$ and on the graph of the desired formation. The latter is defined as follows. Let $d = (d_1, ..., d_N)^T \in \mathbb{R}^{2N}$ be the vector of the offsets. Similar to graph $\mathcal{G}(p)$ we define the graph of the desired formation $\mathcal{G}(d) = (\mathcal{V}(d), \mathcal{E}(d))$ which is composed of vertices in positions d_is and edges given by $e_{i,j} \in \mathcal{E}(d)$ if $w(d_i, d_j) = 1 \iff \|d_i - d_j\| \le \rho$. To ensure an information flow between all agents in the desired formation we introduce the following assumption.

Assumption A1: The graph $\mathcal{G}(d)$ associated to the desired (targeted) formation is connected.

Theorem III.1. *Let $\mathcal{G}(d)$ be the graph associated to the desired formation and let ρ be the transmission range. Let assumption **A1** be satisfied.*

If initial positions $p(t_0)$ are such that $\|p_i(t_0) - p_j(t_0)\| \le \rho$ holds for all pairs $(v_i, v_j) \in \mathcal{E}(d)$, then at each step, the optimization problem in (9) *has a non-zero solution. Moreover, solutions of system* (5) *converge to consensus when the optimal transmission indicators h_k^* defined by* (5) *are applied.*

Proof: The first step of our proof is to show the existence of a feasible solution, and the second step is to study convergence to the desired formation. Lyapunov function $V(x)$ defined in (7) is used in both steps of the proof.

To prove the existence of a feasible solution to problem (9), it is enough to show that there is at least one configuration that gives a feasible solution. Let the assumptions of the theorem be satisfied at t_k. Then, to prove the existence of a feasible solution, it is enough to prove the existence of parameters $h_{k,l}$ such that $\dot{V} \le 0$ over the interval $[t_k, t_{k+1})$ and the connectivity constraint is satisfied at the moment t_{k+1}.

Taking the derivative of function V along solutions of (5) over the interval $[t_k, t_{k+1})$ we obtain

$$\begin{aligned}\dot{V} &= \sum\nolimits_{i,j=1}^{N} (x_i - x_j)^T (\dot{x}_i - \dot{x}_j) \\ &= \sum\nolimits_{i,j,l=1}^{N} h_{k,l} w(p_l, p_i)(x_i - x_j)^T (x_l - x_i) \\ &- \sum\nolimits_{i,j,l=1}^{N} h_{k,l} w(p_l, p_j)(x_i - x_j)^T (x_l - x_j) \\ &= 2\sum\nolimits_{i,j,l=1}^{N} h_{k,l} w(p_l, p_i)(x_i - x_j)^T (x_l - x_i)\end{aligned}$$

The last equality is obtained from the dynamics equation (5) and the interchange of variables i and j in the second term. Next, using equalities $w(p_l, p_j) + [1 - w(p_l, p_j)] = 1$ and $2ab = (a + b)^2 - a^2 - b^2$, we get

$$\begin{aligned}\dot{V} = &- \sum\nolimits_{i,j,l=1}^{N} h_{k,l} w(p_l, p_i) w(p_l, p_j) \|x_i - x_j\|^2 \\ &- \sum\nolimits_{i,j,l=1}^{N} h_{k,l} w(p_l, p_i)[1 - w(p_l, p_j)] \|x_i - x_l\|^2 \\ &- \sum\nolimits_{i,j,l=1}^{N} h_{k,l} w(p_l, p_i)[1 - w(p_l, p_j)] \|x_j - x_i\|^2 \\ &+ \sum\nolimits_{i,j,l=1}^{N} h_{k,l} w(p_l, p_i)[1 - w(p_l, p_j)] \|x_j - x_l\|^2\end{aligned}$$

Since in the expression above only the last term is positive, then to guarantee that $\dot{V} \le 0$ during the super-frame k, it is enough to find a configuration h_k that allows us to dominate the last term by the other terms.

Assume that at super-frame k, there is only one transmitting agent[1] l_{0_k} and therefore $h_{k,l_0} = 1$ and $h_{k,l} = 0$ for $l \ne l_0$. Notice that in this case $\dot{V}$ verify the following inequality:

$$\begin{aligned}\dot{V} \le &- \sum\nolimits_{i,j=1}^{N} h_{k,l_{0_k}} w(p_{l_{0_k}}, p_i)[1 - w(p_{l_{0_k}}, p_j)] \|x_j - x_i\|^2 \\ &+ \sum\nolimits_{i,j=1}^{N} h_{k,l_{0_k}} w(p_{l_{0_k}}, p_i)[1 - w(p_{l_{0_k}}, p_j)] \|x_j - x_{l_{0_k}}\|^2\end{aligned}$$

By choosing transmitting agent as $l_{0_k} = \text{argmin}_{l \in \mathcal{I}_N} \sum_{j=1}^{N} [1 - w(p_l, p_j)] \|x_j(t_k) - x_l(t_k)\|^2$ we guarantee that the first term in the $\dot{V}$ inequality dominates the second term. Therefore, we get $\dot{V} \le 0$. The condition $\dot{V} = 0$ is achieved only if $x_i = x_j$ for $i, j \in \mathcal{I}_N$.

At the same time, from the equation (5), for any $t \in [t_k, t_{k+1})$, we get

$$\begin{aligned}p_i(t) - p_{l_{0_k}}(t) = e^{-(t-t_k)} &\left(p_i(t_k) - p_{l_{0_k}}(t_k)\right) \\ &+ \left(1 - e^{-(t-t_k)}\right)\left(d_i - d_{l_{0_k}}\right)\end{aligned}$$

Then for any $(v_i, v_j) \in \mathcal{E}(d)$, such that $\|d_i - d_j\| \le \rho$ and $\|p_i(t_k) - p_j(t_k)\| \le \rho$, using the triangular inequality we obtain that $\|p_i(t) - p_j(t)\| \le \rho$ for any $t \in [t_k, t_{k+1})$. This property guarantees that the graph $\mathcal{G}(p(t)))$ remains connected during this interval and since k is arbitrary, the connectivity property of $\mathcal{G}(p(t)))$ is preserved for all $t \ge 0$. Thus we showed the existence of a non-zero feasible solution.

The optimization algorithm (9) provides an optimal selection of transmitters that minimizes the value of the cost function $V(x)$ at time t_{k+1} while satisfying the constraints of connectivity preservation and limited network resources. The existence of such an optimal solution is guaranteed from the existence of a single-transmitting-agent feasible solution. Let such an optimal selection of transmitters given by (9) be applied to the system (5) and denote by $V_{opt}(x(t))$ the value of the function $V(x)$ along trajectories based on this selection. Then at instants t_k, values of function $V_{opt}(x(t))$ are upper bounded by a strictly decreasing function $V(x(t))$ that corresponds to the singe-transmitting-agent solution. Therefore, the vehicles converge to the desired formation.

The connectivity condition given in Theorem III.1 can be interpreted as follows. If two agents are supposed to be in each other's transmission range in the desired formation, they must be in each other's transmission range at the initial time. In practical applications, this constraint is easily satisfied since vehicles positions in the formation are chosen such that a minimal maneuver is required.

IV. Simulation results

In this section, we consider a vehicular network composed of $N = 15$ vehicles. The desired configuration is given in the top-right plot in Fig. 3. This configuration is defined by the offsets $d_1 = (0, 0)^T, d_2 = (-2, 2)^T, d_3 = (-2, -2)^T, d_4 = (-4, 0)^T, ..., d_{14} = (-18, 2)^T, d_{15} = (-18, -2)^T$. At time t_0,

[1] Notice that such a choice guarantees that the constraint of limited network resources is satisfied, since $M \ge 1$.

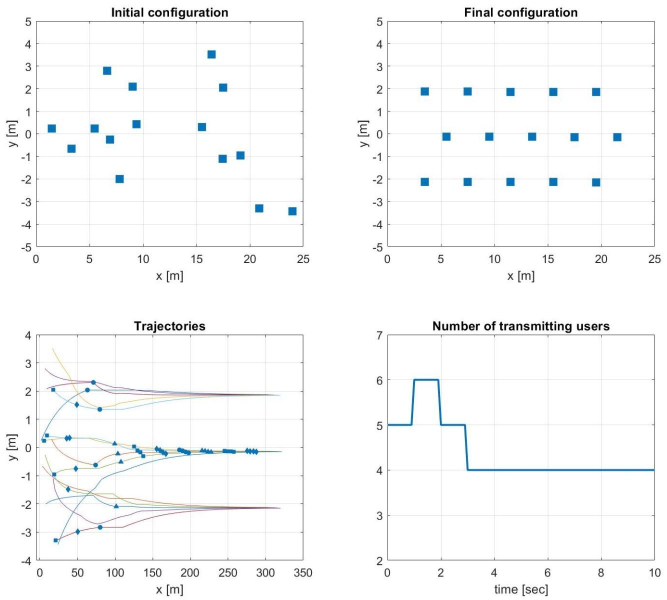

Fig. 3. Vehicles positions at (top-left) the initial time and (top-right) the final time. (bottom-left) Trajectories of the vehicles and transmitting agents at each step. (bottom-right) the number of transmitting users during the simulation. The vehicles converge to the desired formation.

the vehicles distributed randomly in a 2-dimensional space such that their distribution satisfies the condition given in Theorem III.1. The initial configuration is given in the top-left plot in Fig. 3.

The frame duration is equal to $T_f = 10ms$ and the resources are used by the same users for $q = 100$ consecutive frames. Thus, the duration of the super-frame is equal to $T_p = 1s$. We suppose that the number of available resources is $M = 4$. The transmitting range is given by $\rho = 4$. The reference trajectory is $\dot{p}_i(t) = u_{ref}$, with $u_{ref} = (30, 0)^T$.

At instants $t_k = 0s, 1s, 2s....$ the proposed algorithm is executed at the base station and the resources are assigned to the transmitting agents selected as the optimal solution given by (9). In this example, problem (9) is solved using the non-linear Branch-and-Bound algorithm. The simulation results are plotted in Fig. 3. The bottom-left plot shows the vehicles trajectories in 2-dimensional space. The markers on the trajectories correspond to the positions of the transmitting agents at the beginning of each step, where different markers correspond to different steps. The bottom-right plot shows the number of transmitting users at each period.

Figure 3 shows that the vehicles achieve the desired formation. Moreover, the network resources are used efficiently. Indeed, while agents are tending the desired formation, the number of transmitting agents is higher than the number of available resources $M = 4$. Thus, the proposed algorithm benefits from the vehicles positions to allow multiple users to use the same resource while guaranteeing an interference-free communication.

V. Conclusion

In this paper, we proposed a joint design method for formation control and communication strategy for connected vehicles over a 5G network. We gave a characterization of the domain of the initial condition from which it is guaranteed to converge to the desired formation. This domain is related to the targeted formation. Future works include the robustness analysis of the proposed scheme with respect to communication errors such as packet losses or transmission delays.

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